Nonequilibrium Dynamic Phase transitions in ferromagnetic systems: Some new phenomena

Muktish Acharyya

Department of Physics, Krishnanagar Government College,
PO-Krishnanagar, Dist-Nadia, PIN-741101, West-Bengal, India.
E-mail:muktish@vsnl.net

The thermodynamical behaviours of ferromagnetic systems in equilibrium are well studied. However, the ferromagnetic system, far from equilibrium, became an interesting field of research in last few decades. The ferromagnetic systems in the presence of a steady magnetic field are also studied by using standard tools of equilibrium statistical physics. The ferromagnet in the presence of time dependent magnetic field, shows various interesting phenomena, explored very recently. An usual response of a ferromagnet in presence of a sinusoidally oscillating magnetic field is the hysteresis. Apart from this hysteretic response, the nonequilibrium dynamic phase transition is a very interesting phenomenon. In this chapter, the nonequilibrium dynamic phase transitions, in model ferromagnetic systems in presence of time dependent magnetic field, are discussed. For this kind of nonequilibrium phase transition one cannot employ the standard techniques of equilibrium statistical mechanics. The recent developments in this direction are mainly based on numerical simulation (Monte Carlo). The Monte Carlo simulation, of kinetic Ising model in presence of sinusoidally oscillating (in time but uniform over space) magnetic field, is extensively performed to study the nonequilibrium dynamic phase transition. The temperature variations of dynamic order parameter, dynamic specific heat, dynamic relaxation time etc. near the transition point are discussed. The appearance of a dynamic length scale and a dynamic time scale and their behaviours near the transition point are also discussed. All these studies indicate that this proposed dynamic transition is a nonequilibrium thermodynamic phase transition. The disorder (quenched) induced zero temperature (athermal) dynamic transition is studied in random field Ising ferromagnet. The dynamic transition in the Heisenberg ferromagnet is also studied. The nature of this transition in Heisenberg ferromagnet depends on the anisotropy and the polarisation of the applied time varying magnetic field. The anisotropic Heisenberg ferromagnet in presence of elliptically polarised magnetic field shows multiple dynamic transitions. This multiple dynamic transitions in anisotropic Heisenberg ferromagnet are discussed here. Recent experimental evidences of dynamic transitions are also discussed very briefly.

1. Introduction:

The ferromagnetic system in statistical equilibrium gives rise to well-known ferro-para phase transition. However, the ferromagnetic system, in the presence of a time varying external magnetic field, remains far from thermodynamical/statistical equilibrium. This type of system became an interesting object of research over last two decades. Particularly, if the time varying externally applied magnetic field is sinusoidally oscillating (in time but uniform over the space), it yields two major responses of the ferromagnetic systems, i.e. (i) dynamic hysteresis (ii) dynamic phase transition. The dynamic hysteretic responses of Ising-like systems are already discussed in a recent review article [1]. But very recently, the another response, i.e., the dynamic transition has drawn much attention of researchers. It has several interesting and important (in the context of nonequilibrium phase transition) aspects and can provide a simple example of nonequilibrium transition. In the earlier review [1], the dynamic transition was also discussed in brief. However, in last five years a considerable amount of research was done on it (in Ising model) which was not reviewed yet. Very recently, the study of the dynamic transitions in Heisenberg model gives several interesting features which are not possible to observe in Ising models. Keeping this in mind, I have tried to review the recent developements in this field.

This review is mainly divided into two parts. The first part is devoted to the dynamic transitions in Ising model. Starting from the example of primary observation of dynamic transition in the meanfield dynamical equation, the justification of calling the dynamic transition was argued. The thermal fluctuation incorporated dynamic transition in kinetic Ising model reviewed. The recent developements in last five years

(after the publication of the earlier review [1]) are elaborately mentioned. The second part is written for the Heisenberg model with mentioning the very recent works on it. Lastly, a small paragraph was written for the experimental developements.

The paper is organised as follows: in the next section a short introduction is written about what a dynamic transition is and when it should be called true dynamic transition. In third section, the dynamic transition in Ising ferromagnet is discussed. Different aspects of dynamic transitions (mainly its thermodynamic natures) and its relationship with the stochastic resonance are reviewed. In this review the analytic formulation of dynamic transition and the *athermal* dynamic transition. In the fourth section, the dynamic transition in Hesenberg ferromagnet is discussed. This section gives a review of very recent (last 1-2 years) works on classical vector spin models. The results available so far, based on experimental observations are reviewed in fifth section. The chapter ends with a summary given in sixth section.

2. What is dynamic transition ?

If the temperature of a ferromagnetic sample increases (in absence of external magnetic field), the sponteneous magnetisation vanishes at a particular temperature (Curie temperature) and the transition occurs from an asymmetric (ferromagnetic) phase to a symmetric (paramagnetic) phase. This is a very well known phenomenon and the equilibrium symmetry breaking ferro-para transition is also well studied. But any nonequilibrium transition of any ferromagnetic sample in the presence of time dependent magnetic field was unexplored even before 1990!! Tome and Oliviera [2] first observed a prototype of nonequlibrium dynamic transition in the numerical solution (by fourth order Runge-Kutta method) of mean field equation of motion

$$\frac{dm}{dt} = -m + \tanh\left[\frac{m(t) + h(t)}{T}\right] \tag{1}$$

of the classical one component (m) ferromagnetic model in presence of a magnetic field varying sinusoidally ($h(t) = h_0 \cos(\omega t)$) in time. Here, the time averaged magnetisation $Q = \frac{1}{\tau}\oint m(t)dt$ ($\tau = 2\pi/\omega$ is the time period of the oscillating field) over a full cycle of the oscillating field plays the role of order parameter for this type of proposed nonequlibrium dynamic transition. Tome and Oliveira [2] found that Q becomes zero (dynamically disordered phase) from a nonzero (dynamically ordered phase) value at a finite temperature T (which also depends on the value of amplitude h_0 of oscillating field). They sketched a dynamic phase boundary in $h_0 - T$ plane. For the higher values of h_0 the transition was found to be of discontinuous type and that becomes continuous for lower values of h_0. They [2] also located a tricitical point (TCP) on the phase boundary which separates the nature (discontinuous/continuous) of these transitions.

This phenomenon can be explained by considering the system initially kept in one well of a Landau type double well potential. Depending on the temperature, a definite amount of magnetic field is necessary to bring the system from one well to another. If the amplitude of the applied oscillating magnetic field is less than the required amount, the system oscillates in one well (where it was initially). In this situation, the magnetisation does not changes its sign. As a result, the magnitude of the time averaged magnetisation is nonzero ($Q \neq 0$). If one see the plot of $m(t) - h(t)$, it is asymmetric about $m(t) = 0$ line. This gives rise to a dynamically ordered and asymmetric phase. As the temperature increases, the height of the barrier between two wells decreases and to push the system from one well to another, less amount of magnetic field is necessary. As a result, the magnetisation can change its sign for this amount of field. Consequently, the time averaged magnetisation over a full cycle becomes zero ($Q = 0$) when $m(t) - h(t)$ plot is symmetric. So, we get the symmetry broken dynamic phase transition, where the transition temperature decreases as the value of magnetic field increases. This can qualitatively explain the phase boundary of dynamic phase transition. Fig. 1 illustrates this symmetry breaking associated to the dynamic transition.

But, can we really call this transition a true dynamic transition ? A true dynamic transition will be such that this transition should disappear in the static limit !! Let us examine logically,

what happens in the static (infinitesimally small frequency) limit. One can simulate this situation by varying the magnetic field with infinitesimally small frequency. It should ne noted here (in eqn.1) that the dynamics of the one component magnetisation (m) is purely deterministic. So, at a particular temperature, to bring the system from one well to another definite amount of field should be applied, irrespective of the rate of achieving the required amount. The system will wait untill it gets the required value of the field. If the frequency is very small, it will wait for very long time. But unless it gets the required value of field it will not go to another well. There is no stochasticity involved or any noise is present in the equation of motion which can push the system towards another well, irrespective of value of the applied field. So, obviously, in the zero frequency limit a so called dynamic transition will be observed in the case of meanfield study. Just by this argument one can immediately conclude that the kind of transition described above cannot be truely dynamic in nature!!

After realising this, the researchers, interested in this field, tried to study the true dynamic transition in ferromagnetic model systems incorporating the thermal fluctuations as the source of noise or stochasticity, keeping in mind that this stochasticity will help to push the system into another well. Rao, Krishnamurthy and Pandit [3] and Dhar and Thomas [5] tried to observe this perfectly dynamic transition in N-vector model in the $N \rightarrow \infty$ limit. They [3, 5] also tried to observe this in the kinteic Ising model by Monte Carlo (MC) simulation. At the same time, Lo and Pelcovits [4] studied the kinetic Ising model in the presence of sinusoidally oscillating magnetic field by Monte Carlo simulation. But, unfortunately both failed to observe the dynamic transition and to draw the phase boundary. Being motivated from these studies; extensive MC studies [5, 6, 7, 8, 9, 10, 12, 14, 15, 16] were performed in the kinetic Ising model in oscillating magnetic field and the recent developments in this direction are discussed in details in the next section.

Although the numerical solution of mean-field equation (Eqn. 1) cannot provide for a true dynamic transition, it has one importance. The linearised equation is exactly solvable and can be used to get some qualitative features of dynamic transition analytically.

3. Dynamic phase transitions in Ising ferromagnet:

(a) Model and simulation

To investigate the true dynamic phase transitions (incorporating the fluctuations) one simple choice may be the kinetic Ising model in presence of sinusoidally oscillating (in time but uniform over the space) magnetic field studied by Monte Carlo simulation. For this choice, one can take the following Hamiltonian,

$$H = -J\Sigma_{<ij>}S_iS_j - h(t)\Sigma_i S_i \tag{2}$$

where, the first term represents the spin-spin Ising type ferromagnetic interaction. $S_i(\pm 1)$ is Ising spin at i-th site of the lattice, $J(> 0)$ is nearest neighbour ferromagnetic interaction strength. The second term represents the spin-field interaction. $h(t) = h_0\cos(\omega t)$ is applied oscillating magnetic field. h_0 is the amplitude and $\omega(= 2\pi f)$ is the angular frequency of the oscillating field. The boundary condition is periodic in all sides of the lattice.

Due to the presence of second term in the Hamiltonian, the system always remain far from equilibrium. However, the dynamical evolution of the system can be studied by Metropolis algorithm [17].

The studies on Ising systems in the presence of an oscillating magnetic field have been made mostly by employing the Monte Carlo method using Metropolis single spin flip dynamics[17]. Starting from an (random or from perfectly ordered) initial configuration of spins, the spin state $S_i(t)$ at any site i and in any time t for a fixed temperature T is updated (sequentially or randomly) with the following probability function [17]

$$W(S_i \rightarrow -S_i) = \mathrm{Min}[1, \exp(-\frac{\Delta H}{K_BT})] \tag{3}$$

where $\Delta H = 2S_i[\Sigma_j S_j(t) + h(t)]$, is the change in energy due to spin flip, K_B is Boltzmann constant and T is the temperature. First the instantaneous response magnetisation per lattice site at time t is easily calculated: $m(t) = \frac{1}{N}\Sigma_i S_i(t)$, where N is the total number of spins in the lattice ($N = L^d$ if one considers a d-dimensional hypercubic lattice of linear size L). N such spin updates is defined as one MC step per spin(MCSS). This is the unit of time in this simulational study. After that, one can study the dynamical response of the system. In this chapter, only the dynamic transitions, out of various kinds of dynamical responses, will be discussed. The dynamical order parameter (as defined in ref[2]) can be calculated as $Q = \frac{1}{\tau}\oint m(t)dt$, where $\tau = (1/f)$ (f is linear frequency) is the time period of the applied oscillating magnetic field. The magnitude of the magnetic field is measured in the unit of J and the temperature is measured in the unit of J/K_B. It should be mentioned here that the results are independent of the dynamics (Metropolis, Glauber etc.) employed to study the dynamic transitions.

(b) Dynamic transition:

The value of stabilised Q is calculated [6] for fixed values of T, h_0 and f. It was observed that for a fixed frequency f, the dynamic order parameter Q is nonzero for lower values of T and h_0 and it would vanish for higher values of T and h_0. For lower values of temperature and the field amplitudes the system is dynamically ordered and it loses its dynamical ordering for higher values of T and h_0. This dynamical order-disorder transition is associated to the breaking of symmetries of $m-h$ loop . For lower values of T and h_0, the magnetisation oscillates asymmetrically (around $m = 0$ line) which gives rise to asymmetric $m-h$ loop (resides asymmetrically in $m-h$ plane) (see Fig.1). In this case $Q \neq 0$, and this phase is a symmetry broken phase. When T and h_0 become large, the magnetisation oscillates symmetrically (around $m = 0$ line). This gives rise to symmetric $m-h$ loop (resides symmetrically in $m-h$ plane) and consequently $Q = 0$. This phase is called symmetric phase. In this regard, one may call that this dynamic transition is associated to a dynamical symmetry breaking.

Extensive Monte Carlo (MC) simulation was performed [1] to study this dynamic transition. MC simulation was done in Ising ferromagnet in two and three dimensional hypercubic lattices. The dynamic transition was observed and the dynamic phase boundary was drawn in $h_0 - T$ plane taking frequency as a parameter. The dynamic phase boundary in both the dimensions (two and three) are qualitatively similar in nature. Along the phase boundary the dynamic transition occurs at higher temperatures for lower values of field amplitudes. The high temperature (low field) transitions are continuous and the low temperature (high field) transitions are discontinuous. A point (marked TCP in figure 2) separating these two types of transitions is called a tricritical point. Another important thing should be noted in this dynamic phase boundary is, the variation of the phase boundary with respect to the frequency of the driving field. The figure (Fig. 2) shows, the phase boundary shrinks inward as the frequency decreases. As an extrapolation of this senario, one may conclude that in the static limit (zero frequency limit) the dynamic transition disappears. This is the important significance of *true* dynamic transition.

The dynamic transition is, in fact, a manifestation of the coercivity property (one of the important features of hysteresis)[13, 21]. In the $Q \neq 0$ phase, the $m-h$ loop is not symmetric about the field axis and lies in the upper (or lower) half of the $m-h$ plane depending on their initial magnetisation. A minimum magnitude of external and opposite magnetic field (coercive field) is required to change the sign of the magnetisation for complete reversal within the time period of the oscillating magnetic field. This magnitude of the coercive field depends on the temperature T. The magnitude of coercive field increases as the temperature decreases. In the case of sinusoidally oscillating magnetic field, for a transition to a $Q = 0$ phase from $Q \neq 0$ phase, the field amplitude should be at least of the order of coercive field depending upon the temperature T. So, in a sense, the dynamic phase boundary (in the low frequency limit) is the coercive field variation with respect to the temperature. Since, $h(t) = h_0\cos(\omega t)$ and $|h(t)| \leq h_0$, the phase boundary, is the upper bound of the coercive field variation with respect to the temperature T. However, the difference of this upper bound increases

with increasing frequency, because of the effectice relaxational lag (τ_{eff}, time lag of magnetisation with respect to the applied field). The effective relaxational lag of the magnetisation arises due to the intrinsic relaxation time of the system. In fact, the tricritical point $T_d^{TCP}(h_0, \omega)$ on the phase boundary appears because of the system's failure to relax within the time period $2\pi/\omega$ of the oscillating field. The intrinsic relaxation time in the ferromagnetic phase decreases with lowering of temperature and below $T_d^{TCP}(h_0, \omega)$, $\tau_{eff} \leq 2\pi/\omega$ (equality at $T = T_d^{TCP}$). So, that the magnetisation changes sign (from $+m$ to $-m$) abruptly and consequently Q changes from a value very near to unity to zero discontinuously. This indicates the TCP should decrease with higher frequency. At zero temperature, the transition is completely mechanical (purely field driven; without any thermal fluctuation) and can only be discontinuous one. Above $T_d^{TCP}(h_0, \omega)$, the thermal fluctuation win-over and determine the continuous nature of the transition. There is a controversy regarding the existence of TCP. The details of studies on TCP will be discussed in the subsection where the relation between dynamic transition and stochastic resonance, is discussed.

It should be mentioned here that another type of dynamic phase transition was studied [19] in kinetic Ising model with negative pulsed magnetic field of finite duration. But in this chapter, the dynamic phase transition, only due to sinusoidally oscillating magnetic field, will be discussed.

(c) Is dynamic transition a phase transition ?

After getting dynamic transition and the dynamic phase boundary, following questions naturally arise: *Is dynamic transition a phase transition ? Is there any evidence of the divergence of time scale and length scale at transition point ? What will be the behaviour of 'dynamic specific heat' and fluctuations of dynamic order parameter near the transition point ?* The studies related to the above questions will be discussed in this subsection.

The critical slowing down is an important phenomenon observed in equilibrium transition (ferro-para) which indicates the divergence of time scale (relaxation time) at transition point. In the MC simulation if the initial condition is all spins directed upward and the sinusoidal magnetic field is driving the system, the $m-h$ loop get stabilised after a transient behaviour. As a result, the dynamic order parameter Q has also a transient behaviour.

It has been observed carefully [7] that the dynamic order parameter Q does not acquire the stable value within the first cycle of the oscillating field. It takes several cycles (of the oscillating field) to get stabilised i.e., it shows 'relaxation' behaviour. Starting from the initial (all spins are up) configuration, the Q has been calculated for various number (say n-th) of cycles of the oscillating magnetic field and plotted (inset of Fig.3) against the number of cycles (n). Each value of Q shown here has been obtained by averaging over 100 random Monte Carlo samples. Inset of Fig.3 shows a typical 'relaxation' behaviour of the dynamic order parameter Q. This has been plotted for fixed values of $\omega = 2\pi \times 0.04$, h_0= 1.0 and T= 1.5. It shows that the dynamic order parameter Q is relaxing as the time (number of cycles) goes on. The best fit curve shows that the 'relaxation' is exponential type. So, one can write $Q \sim Q_0 \exp(-n/\Gamma)$, where Γ is the 'relaxation' time which provides the 'time scale' for this prototype of nonequilibrium problem. The physical interpretation of Γ is, the number of cycles required, so that Q becomes $1/e$ times of its initial value (value of Q at starting cycle). From the exponential fitting, the 'relaxation' time (Γ) has been measured. The temperature (T) variation, for fixed values of ω and h_0, of this 'relaxation' time Γ has been studied (in the disordered region of dynamic transition) and displayed in Fig.3. The temperature (T) variation of Γ are shown (Fig.3) for two different values of h_0(= 0.5 and 1.0) and for a fixed value of $\omega = 2\pi \times 0.04$ here. From the figure (Fig.3) it is clear that the relaxation time Γ diverges near the dynamic transition point (where Q vanishes) in the both cases ($h_0 = 0.5$ and 1.0). This is an important study [7] which first indicated that this dynamic transition is associated to a diverging 'time scale'.

An analytical formulation of this critical slowing down (of Q) can be done [7] by solving the linearised mean-field equation of motion for the average magnetisation . In the limit of $h_0 \to 0$ and

$T > 1$, the equation (eqn. 1) can be linearised (i.e., linearising tanh term) as

$$\tau \frac{dm}{dt} = -\epsilon m + \frac{h_0 \cos(\omega t)}{T},$$

where $\epsilon = 1 - 1/T$. The solution of the above equation is

$$m(t) = \exp(-\epsilon t/\tau) + m_0 \cos(\omega t - \phi),$$

where m_0 and ϕ are two constants. The value of the dynamic order parameter Q at $n-$th cycle of the oscillating field is

$$Q = \frac{\omega}{2\pi} \oint m(t)dt = \frac{\omega}{2\pi} \int_{t_{n-1}}^{t_n} m(t)dt,$$

where $t_n = 2\pi n/\omega$. The value of Q, at the n-th cycle, can be written as

$$Q = Q_0 \exp(-\frac{2\pi n \epsilon}{\tau\omega}) = Q_0 \exp(-n/\Gamma)$$

Q_0 is a constant independent of n. The above form shows that Q relaxes exponentially with the number of cycles (n) of the oscillating field. The 'relaxation' time Γ is equal to $\frac{\tau\omega}{2\pi}\epsilon^{-1}$. It should be noted here that the dynamic transition occurs at $T = 1$ in the limit $h_0 \to 0$. So, for $h_0 \to 0$ near the dynamic transition point (where the linearisation holds good) the behaviour of relaxation time is

$$\Gamma \sim \epsilon^{-1} \sim (T - T_d(h_0 \to 0))^{-1}$$

which shows the power law (exponent is unity) divergence of the 'relaxation' time at the dynamic transition point.

(d) Behaviour of dynamic 'specific-heat' near transition point

The total energy averaged over a cycle of the oscillating magnetic field can be written as [7]: $E_{tot} = \frac{1}{\tau} \oint H dt$, where H is the Hamiltonian (equation 2) of the system. The dynamic 'specific heat' C_{tot} can be defined as the temperature derivative of the total energy E_{tot}. Now, if $C_{tot}(= \frac{dE_{tot}}{dT})$ is plotted against the temperature, the plot shows (Fig.4) a very sharp peak at the transition point in believe that it will diverge for infinite system. This behaviour is similar to that observed in well known equilibrium phase transitions. One can detect the dynamic transition and can have an estimate about the transition point from the temperature variation of the response like dynamic 'specific heat'.

In the equilibrium phase transition it is well known that the specific heat is related to the fluctuation in energy. What one should expect in the case of this type of nonequlibrium transition ? To have a direct answer to this question the fluctuation in 'energy' is studied as a function of temperature. The fluctuation in total energy is: $\delta E_{tot}^2 = (< E_{tot}^2 > - < E_{tot} >^2)$. Here the symbol $< ... >$ denotes the average over variuos samples (obtained from the different MC samples) If this quantity is plotted against the temperature, it would also get sharply peaked at the transition temperature [8].

(e) Evidence of diverging length scale

The evidence of dynamic correlation length of this type of dynamic transition observed in Ising like extended system was also reported [15]. The dynamic susceptibility (in 2D) is defined as $\chi = L^2[< Q^2 > - < Q >^2]$, keeping in mind that the fluctuation-dissipation theorem holds good [8] for this type of nonequilibrium transition also. This is plotted against $1/R = (2\pi/\omega)/ < \tau(h_0) >$ where $< \tau(h_0) >$ is average lifetime or nucleation time of the system. χ was plotted against $1/R$ in Figure 5 for various values of $L(= 64, 90, 128)$. The figure shows the peak of χ increases in height with increasing system size (L). This clearly indicates the finite size effects in χ and implies the

existence of a divergent length scale associated to the order parameter correlation function near the transition point. It is important to note here that this study was done by varying $1/R$ (keeping T and h_0 fixed) whereas most of the studies on dynamic transition have been done by varying the temperature T (keeping ω and h_0 fixed). However, the results are qualitatively invariant under the choice of tunnable parameter. This study [15] gives an important idea regarding the divergence of 'length scale' at the transition point of this dynamic transition.

(f) Dynamic phase transition and hysteresis
(i) Analytic forms of the loop area and the dynamic correlation near the transition point
In the earlier sections it was discussed that the dynamic transition has a very close resemblance with the magnetic hysteresis. In this section, the relations among the hysteretic loss, the dynamic order parameter and the dynamic correlations are dicussed [9].
The form of the oscillating magnetic field is

$$h(t) = h_0 \cos(\omega t). \tag{4}$$

The dynamic order parameter is defined as

$$Q = \frac{\omega}{2\pi} \oint m(t) dt, \tag{5}$$

which is nothing but the time averaged magnetisation over a full cycle of the oscillating magnetic field. The hysteresis loop area is

$$A = -\oint m dh = h_0 \omega \oint m(t) \sin(\omega t) dt, \tag{6}$$

which corresponds to the energy loss due to the hysteresis. The Dynamic correlation is defined as

$$C_d = < m(t)h(t) > - < m(t) >< h(t) >,$$

where $< .. >$ denotes the time average over the full cycle of the oscillating magnetic field. Since $< h(t) > = 0$, one can write

$$C_d = \frac{\omega}{2\pi} \oint m(t)h(t) dt = \frac{\omega h_0}{2\pi} \oint m(t) \cos(\omega t) dt. \tag{7}$$

The dynamic correlation has another physical interpretation. For the cooperatively interacting spin system, this is the negative of the time averaged spin-field interaction energy (per spin) in d-dimensions ($< E_f >= -\frac{\omega}{2\pi L^d} \oint \sum_i \sigma_i \; h(t) dt$) over a complete cycle of the oscillating field.

In the dynamically disordered ($Q = 0$) phase and near the transition point, the time series of the magnetisation ($m(t)$) can be approximated as a square wave with a phase lag δ with the applied sinusoidal magnetic field. This approximation works well in the low temperature region.

$$m(t) = \begin{cases} 1 & \text{for} \quad 0 < t < \tau/4 + \delta/\omega \\ -1 & \text{for} \quad \tau/4 + \delta/\omega < t < 3\tau/4 + \delta/\omega \\ 1 & \text{for} \quad 3\tau/4 + \delta/\omega < t < 2\pi/\omega, \end{cases} \tag{8}$$

where τ is the time period of the oscillating field and δ is the phase lag between magnetisation $m(t)$ and the magnetic field $h(t) = h_0 \cos(\omega t)$. The value of the hysteresis loop area can easily be calculated as

$$A = 4h_0 \sin(\delta). \tag{9}$$

This form of the loop area was also obtained from the assumption that it is approximately equal to four times the product of coercive field and remanent magnetization (here the remanent magnetisation equal to unity), where the coercive field is identified as $h_0 \sin(\delta)$ (the change in field during the

phase lag). Considering the same form of the magnetisation the dynamic correlation C can also be calculated exactly as

$$C_d = \frac{2h_0}{\pi}\cos(\delta). \tag{10}$$

From the above forms of A and C_d it can be written as

$$\frac{A^2}{(4h_0)^2} + \frac{C_d^2}{(2h_0/\pi)^2} = 1. \tag{11}$$

The above relation shows that the loop area A and the dynamic correlation C_d is elliptically related to each other.

The qualitative nature of the dynamic phase boundarycan be realised by considering the simplified form of the instantaneous magnetisation in the ordered phase. The dynamically ordered region ($Q \neq 0$) can be approximated by considering the following form of the magnetization

$$m(t) = \begin{cases} 1 & \text{for} \quad 0 < t < \tau/4 + \delta/\omega \\ 1 - m_r & \text{for} \quad \tau/4 + \delta/\omega < t < 3\tau/4 + \delta/\omega \\ 1 & \text{for} \quad 3\tau/4 + \delta/\omega < t < 2\pi/\omega. \end{cases} \tag{12}$$

In the above simplified approximation, it was considered (since $Q \neq 0$) that the magnetisation can not jump to the other well, however the value of initial magnetisation is reduced by the amount m_r. In the real situation it has been observed that this well is not fully square (as assumed above in the form of $m(t)$), it has a cusp like (or parabolic) shape. For $m_r = 2$, the above functional form of $m(t)$ will take the form of eqn. 8 and one can get the disordered ($Q = 0$) phase. Taking the above form of magnetisation the dynamic order parameter Q can be calculated as $Q = (2 - m_r)/2$. It may be noted that, in this simplified approximation the dynamic order parameter Q is independent of phase lag δ, which is not observed in the real situation (phase lag shows a peak at the transition point) [9]. However, this simple picture can anticipate the convex (looking from the origin) nature of the dynamic phase boundary. As the temperature increases m_r increases and it also increases as the field amplitude increases. Since m_r increases as h_0 and T increases, in the simplest linearised asumption, one can consider m_r is proportional to the product of h_0 and T. Demanding, $m_r = 2$ for the dynamic transition ($Q = 0$), one can readily obtain $(h_0)_d T_d =$ constant. This equation tells that the dynamic phase boundary will be convex having the shape of rectangular hyperbola. The convex nature of the phase boundary remains invariant even if one assumes that m_r is any increasing function of both T and h_0 (for example, power law; $m_r \sim T^x h_0^y$, in this particular case the equation of the dynamic phase boundary becomes $T_d^x (h_0)_d^y =$ constant, it is easy to see that this gives the convex shape of the dynamic phase boundary). However, this very simple asumption can not describe the entire form of the phase boundary accurately, particularly near the end points ($(h_0)_d = 0$ and $T_d = 0$).

(ii) General relation among Dynamic order parameter, Hysteresis loop area and the Dynamic correlation

From the usual definitions (given in earlier section) of C_d and A, one can write

$$\frac{1}{\sqrt{2\pi}}\left(\frac{2\pi C_d}{\omega h_0} - i\frac{A}{\omega h_0}\right) = \frac{1}{\sqrt{2\pi}}\oint m(t)\exp(-i\omega t)dt,$$

where $m(\omega) = \frac{1}{\sqrt{2\pi}}\oint m(t)\exp(-i\omega t)dt$. So,

$C_d = \frac{h_0\omega}{\sqrt{2\pi}}\mathrm{Re}\left(m(\omega)\right)$

and

$A = -h_0\omega\sqrt{2\pi}\mathrm{Im}\left(m(\omega)\right)$.

The general (complex) form of $m(\omega')$ will be

$m(\omega') = |m(\omega')| \exp(i\phi)$

$m(\omega') = \frac{1}{\sqrt{2\pi}} \left(\frac{4\pi^2 C_d^2}{h_o^2 \omega'^2} + \frac{A^2}{h_0^2 \omega'^2} \right)^{1/2} \exp i \left[-\tan^{-1} \frac{A}{2\pi C_d} \right]$

Note that the phase ϕ of $m(\omega')$ is independent of h_0 and ω. So, Q is related with A and C_d as follows

$$Q = \frac{1}{\tau} \oint m(t) dt$$

$$= \frac{1}{\sqrt{2\pi}\tau} \int d\omega' \oint m(\omega') \exp(i\omega' t) dt$$

$$= \frac{1}{2\pi\tau} \int d\omega' \oint \sqrt{\left(\frac{4\pi^2 C_d^2}{h_o^2 \omega'^2} + \frac{A^2}{h_0^2 \omega'^2} \right)} \mathbf{e}^{\left[i(\omega' t - \tan^{-1} \frac{A}{2\pi C_d}) \right]} dt. \quad (13)$$

Above equation gives the general relationship among Q, A and C_d.

It has been observed that the steady response $m(t)$, to a sinusoidally oscillating magnetic field ($h(t) = h_0 \cos(\omega t)$), is periodic (with phase lag δ) and has the same periodicity ($\tau = 2\pi/\omega$) of the field. So, one can write $m(t)$ in a Fourier series as

$$m(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t). \quad (14)$$

From the usual definitions of Q, A and C_d, it is easy to see that

$a_0 = Q$, $a_1 = 2C_d/h_0$ and $b_1 = A/(\pi h_0)$.

So, one can write

$$m(t) = Q + \frac{2C_d}{h_0} \cos(\omega t) + ... + \frac{A}{\pi h_0} \sin(\omega t) + ... \quad (15)$$

Keeping only the first harmonic terms (ignoring higher harmonics) one can easily express the instantaneous magnetization as

$$m(t) = Q + m_0 \cos(\omega t - \delta) \quad (16)$$

where the amplitude of magnetization is $m_0 = [(2C_d/h_0)^2 + (A/(\pi h_0))^2]^{1/2}$ and the phase lag is $\delta = \tan^{-1}(A/(2\pi C_d))$.

(g) Dynamic phase transition and stochastic resonance

To study the relationship between dynamic transition and the stochastic resonance [18] extensive MC simulations were performed [12] in kinetic Ising model in presence of sinusoidally oscillating magnetic field. The frequency is $f = 0.001$ (kept fixed throughout the study). So, one complete cycle of the oscillating field takes 1000 MCSS (time period $\tau = 1000$ MCSS). A time series of magnetization $m(t)$ has been generated up to 10^6 MCSS. This time series contains 10^3 (since $\tau =$ 1000 MCSS) number of cycles of the oscillating field. The dynamic order parameter Q has been calculated for each such cycle. So, the statistics (distribution of Q) is based on $N_s = 10^3$ different values of Q.

The statistical distribution $P(Q)$ of dynamic order parameter Q and its temperature dependence have been studied [12] closed to the phase boundary to detect the nature[20] of the transition. Fig.6a shows the distributions $P(Q)$ (at fixed value of the field amplitude) for three different values of temperature. Below, the transition the distribution shows only two equivalent peaks centered

around ± 1. Close to the transition point, a third peak centered around zero is developed. As the temperature increases slightly, the strength of the third peak increases in cost of that of two other (equivalent) peaks. Above the transition, only one peak is observed centered around zero. This indicates that the transition is first order or discontinuous.

What is the origin of this kind of first order transition ? To get the answer of this question, the time variation of the magnetization $m(t)$ is studied [12] for several cycles of the oscillating magnetic field $h(t)$, close to the transition. Sometimes, the system likes to stay in the positive well (of the Landau type double well form of the free energy) and sometimes it likes to stay in other. It is obvious that the best time for the system to switch from one well to the other one, is when the value of the field is optimum ("good opportunity") . So, if the system misses one "good opportunity" (first half period of the oscillating field) to jump to the other well it has to wait for a new chance (another full period of the oscillating field). Consequently, it shows that the residence time (staying time in a particular well) can only be nearly equal to an odd integer multiple of the half-period (half of the time period of the oscillating field). This leads to two consequences:

(1) The distribution of the dynamic order parameter Q would be peaked around three values (i) $Q \approx 0$, when the system utilizes "good opportunity" and goes from one well to the other (marked 'A' in Fig.6a), (ii) $Q \approx -1$, when the system misses the "good opportunity" to go from negative well to the positive well and it stays for one (or more) full period in the negative well (marked 'B' in Fig.6a), (iii) $Q \approx +1$, when the system misses the "good opportunity" to go from positive well to negative well and spends one (or more) full period in the positive well (marked 'C' in Fig.6a). As a result, the distribution of Q would give three distinct peaks centered at +1, -1 and 0.

(2) The other consequence of this kind of time variation, of magnetization $m(t)$, is the "stochastic resonance" [18]. This can be detected from the distribution of residence time (the time system spends in a particular well). The distribution (P_r) of residence time (τ_r) will be peaked multiply around the odd integer multiple of half-period. One such distribution is shown in Fig.7. The distribution shows multiple peaks around the odd integer values (500, 1500, 2500, 3500, 4500 and 5500 MCSS) of half-period ($\tau/2$=500 MCSS,of the driving fields). The heights of the peaks decreases exponentially (dotted line in Fig.7) with the peak positions. This is the fingureprint of stochastic resonance [18].

Figure 6 shows the distributions of the dynamic order parameter Q for three different values of the temperature. Here, the field amplitude h_0 is quite low in comparison with that used in the earlier case (Fig.6a). It shows that, in the ordered region, this gives two (equivalent) peaks (Fig.6b) and as the temperature increases these two peaks come close to each other continuously (Fig.6b) and close to the transition (and also above it) (Fig.6b) only one peak (centered around zero) is observed. This feature reveals the continuous or second order transition. Hence, it was proved that a tricritical point would exist which separates the nature (discontinuous/continuous) of the dynamic transition. However, a recent study [16] claims that the existence of TCP [12] is a correct observation but a finite size effect. For small system size the distribution of dynamic order parameter Q shows (Fig.8A) three peaks very close to the transition point. This was observed earlier [12] for small system size. However, the distribution of Q has (Fig. 8B) only two peaks near the transition point for much larger systems revealing only the continuous nature of the transition.

(h) Dynamic phase transition for randomly varying field:

Very recently, an interesting version of this dynamic phase transition has been predicted [10] in a ferromagnetic Ising system when the external field on the system varies in time stochastically. The long time response (magnetisation) of a kinetic Ising system represented by the Hamiltonian (eqn. 2) is studied when the uniform field over the sample $h(t)$ varies randomly in time with a uniform distribution bounded between $+h_0/2$ and $-h_0/2$. In a Monte Carlo simulation study in two dimension, the nature of the response magnetisation (see Fig.9a and Fig.9b) is studied with the dynamic order parameter $Q(= (1/\tau)\int_0^\tau m(t')dt';\ \tau >> 1)$ which is given by the long-time average (over the active duration of the magnetic field) of magnetisation. It was found that Q assumes nonzero values below a phase boundary line in the $h_0 - T$ plane, and vanishes continuously at the

transition boundary (see Fig.9c). Again, the dynamic symmetry breaking transition occurs due to the competing time scales; the relaxation time of the many-body system being larger than the switching time of the random field. Such a dynamic transition is again a nonequilibrium transition, very similar to that for oscillating fields discussed earlier. It may be mentioned that, in a slightly different context, a discrete map version of the mean field equation of motion (eqn. 1) with similar stochastically varying field $h(t)$ was analysed recently by Hausmann and Ruján [11]. The dynamic transition for a randomly varying magnetic field was also studied [10] by solving the mean-field equation (eqn. 1) of motion of average magnetisation.

(i) Athermal dynamic transition in random field Ising model:

The kind of nonequilibrium dynamic transition discussed so far was assisted by thermal fluctuation. An interesting phenomenon, the athermal hysteresis, has been studied [23] recently in random field Ising model. Now one may ask, is there any disorder induced dynamic transition observed at $T = 0$? To investigate this the random field Ising model (in 2D) in presence of oscillating magnetic field was studied [24] at $T = 0$ by MC simulation. A square lattice of linear size L is taken. Each site is labelled by an integer i and carries an Ising spin S_i ($S_i = \pm 1$) which interacts with all its nearest neighbours (spins) with a ferromagnetic interaction strength J. At each site i, there is a local *quenched* random field h_i. The random fields h_i are assumed to be independent and identically distributed random variables with a rectangular probability distribution $P(h_i)$. The random field h_i can take any value from $-w/2$ to $+w/2$ with the same probability. The width of the distribution is w. In addition, there is a uniform (in space) magnetic field $h(t)$ which is varying sinusoidally ($h(t) = h_0 \cos(\omega t)$) in time. The amplitude and the frequency are denoted by h_0 and ω respectively. This kind of model is described by the Hamiltonian

$$H = -J \sum_{<ij>} S_i S_j - \sum_i h_i S_i - h(t) \sum_i S_i, \tag{17}$$

under the periodic boundary condition. For simplicity, the interaction strength J has been set equal to unity throughout the study.

The *zero-temperature* single spin-flip dynamics is specified by the transition rates (W)

$$W(S_i \to -S_i) = \Gamma, \quad \text{if} \quad \Delta E \leq 0 \quad \text{and} \quad W(S_i \to -S_i) = 0, \quad \text{otherwise} \tag{18}$$

where ΔE is the change in energy due to spin flip. In words the algorithm is: never flip the chosen spin if this process would increase the energy and flip otherwise. We have started with all spins are up ($S_i = +1$) as an initial condition and updated the lattice sequentially using the above flipping algorithm. One such full scan over the entire lattice consists a Monte Carlo step per spin (or MCS). The instantaneous magnetisation (per site) $m(t)$ is easily calculated,

$$m(t) = \frac{1}{L^2} \sum_i S_i. \tag{19}$$

After an initial transient period the intantaneous magnetisation $m(t)$ has been found to be stabilised and periodic with the same periodicity of the applied oscillating field. For a particular values of h_0, ω and w the dynamic order parameter $Q(= \frac{1}{\tau} \oint m(t)dt)$ is calculated by averaging over 20 different random disorder (quenched) realisations.

The simulations are performed on a square lattice of linear size $L = 100$ and a particular value of the frequency ($\omega = 0.01 \times 2\pi$) of the oscillating magnetic field. The time is measured in units of Monte Carlo steps per spin or MCS and the values of random field and the oscillating field are measured in units of interaction strength J.

It has been observed numerically that, for fixed values of h_0 and w, the magnetisation becomes periodic (in time) with the same periodicity as the applied sinusoidal magnetic field. For the smaller values of the quenched disorder ($w = 8.0$) and the field amplitude ($h_0 = 0.5$), the magnetisation

oscillates asymmetrically about the zero line i.e., the system remains in a dynamically symmetry broken phase. Consequently, the hysteresis $(m - h)$ loop resides on the upper half plane formed by $h(t)$ and $m(t)$. So, the time averaged magnetisation over a full cycle of the oscillating field, the dynamic order parameter, is nonzero in the symmetry broken phase. By increasing the field amplitude (for $h_0 = 2.0$) keeping w fixed ($w = 8.0$), it was observed that the system acquires a dynamically symmetric phase, i.e., the magnetisation oscillates symmetrically about the zero line. The hysteresis loop is also symmetric. Consequently, the value of the dynamic order paramater Q is zero in this dynamically disordered (symmetric) phase.

In the dynamically disordered phase, the dynamic order parameter Q can be kept at zero in two ways, either by increasing the random field width w for a fixed field amplitude h_0 or vice versa. So, in the plane formed by the field amplitude (h_0) and the width (w) of the quenched disorder (random field), one can think of a boundary line, below which Q is nonzero and above which it vanishes. Figure 10(a) displays such a phase boundary in the $h_0 - w$ plane obtained by Monte Carlo simulation. The nature (discontinuous/continuous) of the transition depends on the value of w and h_0 on the phase boundary line. The transition across the upper part of phase boundary line is discontinuous and it is continuous for the rest part of the boundary. A tricritical point on the phase boundary line separates these natures. Figure 10(b) demonstrates two typical transitions for two sets of values of w and h_0 lying just in the left and right sides of the tricritical point (TCP). In the case of discontinuous transition, the dynamic order parameter Q jumps to a small nonzero value and then vanishes continuously. The uncertainty in the location of the TCP on the phase boundary are shown by the circle enclosing it. It was not yet checked whether this TCP observed here is a finite size effect or not.

4. Dynamic phase transitions in Heisenberg ferromagnet:

(i) Why Heisenberg model ?

Although the nonequilibrium dynamic transition studied in the Ising model, it has some limitations. The Ising model is a special case of general magnetic model [25], for example, the Heisenberg model. The Heisenberg model (with ferromagnetic interactions) having uniaxial anisotropy has some general properties which cannot be found in Ising model. But in the limit of infinite anisotropy, the Heisenberg model can be mapped into Ising model. So, the natural expectation is, the Heisenberg model with uniaxial anisotropy can be studied to have the detailed and general microscopic view and the results can be checked in the limit of infinite anisotropy (which will give the results in Ising model). In this case of dynamic transitions, mainly in the magnetic model system in presence of a magnetic field oscillating sinusoidally in time, the Heisenberg model can serve a better role than an Ising model. It would be quite interesting to know the dynamic response of uniaxially anisotropic Heisenberg model in presence of a magnetic field applied in different directions. On the other hand, there is another advantage. The results obtained in the Ising model is well established [1]. These results can be used to check the results obtained in Heisenberg model by approaching the limit of infinite anisotropy. This prompted to study the dynamic transition in Heisenberg model with uniaxial and single-site anisotropy. Recently, the dynamic transition was studied [31] in the uniaxially anisotropic ferromagnetic Heisenberg model and very interestingly it was observed that the dynamic symmetry of the order parameter component (along the anisotropy direction) can be broken in presence of a magnetic field applied along the direction which is perpendicular to the direction of anisotropy. This transition was named as *off-axial* transition. The transition is found to be continuous and the transition temperature increases as the strength of anisotropy increases.

So, the questions naturally arise what would be the difference in the dynamic transitions in presence of a field applied only along the direction of anisotropy ? How the symmetry breaking takes place ? What would be the nature (continuous or discontinuous) of the transition ? More interestingly, what would happen in infinite anisotropic case and in the Ising case ? To get the answers

of these questions, the researchers studied the dynamic phase transition in classical vector spin models. The dynamical phase transition in anisotropic XY ferromagnet in an oscillating magnetic field is studied recently [26] by solving the time dependent Ginzburg-Landau equation. Very recently, it was observed theoretically [27] that the symmetry of the vector spin model can be tailored by applying oscillating magnetic field. For example [27], depending on the frequency and amplitude of the field the Heisenberg ferromagnet can behave like XY ferromagnet. The dynamic transitions in presence of the axial field (i.e., the magnetic field applied only along the direction of anisotropy) and the off-axial field (i.e., the magnetic field applied only along the direction which is perpendicular to the direction of anisotropy) are studied [31] by Monte Carlo simulation using Metropolis rate. Also, a comparison between axial and off-axial transitions has been made and the results (in the limit of infinite anisotropy) for both cases are compared with that observed in the Ising model. By the application of polarised magnetic field the multiple dynamic transitions were observed [32] in anisotropic ferromagnetic Heisenberg model.

(ii). The description of the model

The Hamiltonian of a classical anisotropic (uniaxial and single-site) Heisenberg model with nearest neighbour ferromagnetic interaction in the presence of a magnetic field can be written as

$$H = -J \sum_{<ij>} \vec{S}_i \cdot \vec{S}_j - D \sum_i (S_{iz})^2 - \vec{h} \cdot \sum_i \vec{S}_i, \tag{20}$$

where $\vec{S}_i[S_{ix}, S_{iy}, S_{iz}]$ represents a classical spin vector of magnitude unity situated at the i-th lattice site. So, $S_{ix}^2 + S_{iy}^2 + S_{iz}^2 = 1$ is an equation of a unit sphere. Classical spin means, this spin vector can be oriented in any direction in the vector spin space. $J(> 0)$ is the uniform nearest neighbour strength of the ferromagnetic interaction. The factor D in the second term is the strength of uniaxial (z here) anisotropy favouring the spin to be aligned along the z-axis. The last term is the spin-field interaction term, where $\vec{h}[h_x, h_y, h_z]$ is the externally applied magnetic field (uniform over the space). When the magnetic field is applied only along the α - direction, the magnetic field component h_α (may be any one of x, y and z) is oscillating sinusoidally in time and can be written as $h_\alpha(t) = h_\alpha^0 \cos(\omega t)$, where h_α^0 and ω are the amplitude and angular frequency ($\omega = 2\pi f$; f is frequency) of the oscillating field respectively. Magnetic field $|\vec{h}|$ and strength of anisotropy D are measured in the unit of J. The model is defined in a simple cubic lattice of linear size L with periodic boundary conditions applied in all the three directions.

(iii) The Simulation technique

The model, described above, has been studied extensively by Monte Carlo simulation using the following algorithm [30]. Initial configuration is a random spin configuration. Here, the algorithm used, can be described as follows. Two different random numbers r_1 and r_2 (uniformly distributed between -1 and 1) are chosen in such a way that $R^2 = (r_1^2 + r_2^2)$ becomes less than or equal to unity. The set of values of r_1 and r_2, for which $R^2 > 1$, are rejected. Now, $u = \sqrt{1 - R^2}$. Then, $S_{ix} = 2ur_1$, $S_{iy} = 2ur_2$ and $S_{iz} = 1 - 2R^2$.

Starting from an initial random spin configuration (corresponding to high temperature configuration) the system is slowly cooled down. At any fixed temperature T (measured in the unit of J/K_B) and field amplitude h_α^0 (measured in the unit of J) a lattice site i has been chosen randomly (random updating). The value of the spin vector at this randomly chosen site is $\vec{S}_i$ (say). The energy of the system is given by the Hamiltonian (equation 1) given above. Now, a test spin vector $\vec{S}_i'$ is chosen randomly (described by the algorithm above). For this choice of spin vector at site i the energy will be $H' = -J \sum_{<ij>} \vec{S}_i' \cdot \vec{S}_j - D \sum_i (S_{iz}')^2 - \vec{h} \cdot \sum_i \vec{S}_i'$. The change in energy, associated to this change in direction of spin vector from $\vec{S}_i$ to $\vec{S}_i'$, is $\Delta H = H' - H$. Now, the Monte Carlo method [25, 20] will

decide how far this change is acceptable. The probability of the change is given by Metropolis rate [25, 20] (used here) $W(\vec{S}_i \rightarrow \vec{S}_i') = \mathrm{Min}[1, \exp(-\Delta H/K_B T)]$. This probability will be compared with a random number R_p (say) between 0 and 1. If R_p does not exceed W, the move (the change $\vec{S}_i \rightarrow \vec{S}_i'$) is accepted. In this way the spin vector $\vec{S}_i$ is updated. L^3 such random updates of spins, defines one Monte Carlo step per site (MCSS) and this is considered as the unit of time in this simulation. The linear frequency ($f = \omega/2\pi$) of the oscillating field is taken 0.001 and was kept constant throughout this simulational study. So, 1000 MCSS is required to get one complete cycle of the oscillating field and consequently 1000 MCSS becomes the time period (τ) of the applied oscillating magnetic field. To calculate any macroscopic quantity, like instantaneous magnetisation components, the following method was employed. Starting from an initially random configuration (which corresponds to a high temperature phase) the system is allowed to be stabilised (dynamically) up to 4×10^4 MCSS (i.e., 40 complete cycles of the oscillating field) and the averages of various physical quantities are calculated from further 4×10^4 MCSS (i.e., averaged over further 40 cycles of the oscillating field). This is quite important to get stable hysteresis loop and it is checked that the number of MCSS mentioned above is sufficient to get stable dynamic phase. Here the total length of this simulation for one fixed temperature T is 8×10^4 MCSS (which produces 80 complete cycles of the oscillating field). Then the system is slowly cooled down (the value of the temperature T has been reduced by small interval) to get the values of the statistical quantities in the low temperature ordered phase. Here, the last spin configuration obtained at the previous temperature is used as the initial configuration for the new temperature. The CPU time required for 8×10^4 MCSS is approximately 25 minutes on an Intel-Pentium-III processor.

(iv) Off-axial dynamic transition

The linear size of the system L has been taken equal to 20. The instantaneous magnetisation components (per lattice site) $m_x = \sum_i S_i^x/L^3, m_y = \sum_i S_i^y/L^3, m_z = \sum_i S_i^z/L^3$ are calculated at each time in presence of magnetic field. The time averaged (over a full cycle of the oscillating field) magnetisation components (the dynamic order parameter components) $Q_x = \frac{1}{\tau}\oint m_x dt, Q_y = \frac{1}{\tau}\oint m_y dt$ and $Q_z = \frac{1}{\tau}\oint m_z dt$ are calculated by integrating (over the complete cycle of the oscillating field) the instantaneous magnetisation components. The total (vector) dynamic order parameter is expressed as $\vec{Q} = \hat{x}Q_x + \hat{y}Q_y + \hat{z}Q_z$.

In this paper, two kinds of dynamic transitions were studied and compared. The *axial* transition means the dynamic order parameter component Q_z becomes zero from a nonzero value at a finite temperature (the transition temperature) in presence of a magnetic field $\vec{h}[0, 0, h_z]$ applied only along the direction which is *parallel* to the direction of anisotropy. Since the uniaxial anisotropy has been taken along the z-direction, in this case, the direction of magnetic field has only nonzero z-component. The *off-axial* transition [31] is the transition in presence of a magnetic field $\vec{h}[h_x, 0, 0]$ applied only along the direction which is *perpendicular* to the direction of anisotropy. In this case, the direction of the magnetic field has only nonzero x-component.

In the case of *axial* transition, the instantaneous magnetisation components are calculated at any fixed temperature T, strength of anisotropy D and amplitude of axial magnetic field h_z^0. The time eliminated plot of $m_z - h_z$ gives the axial hysteresis loop. It was observed that at high temperature ($T = 2.2$) the axial hysteresis loop $m_z - h_z$ is symmetric (symmetric means the loop is distributed about h_z axis in such a way that the total z-component of magnetization, over a complete cycle of field, vanishes) (fig.11a). As a result $Q_z = 0$. And at low temperature ($T = 1.0$) the $m_z - h_z$ loop becomes asymmetric ($Q_z \neq 0$) (fig.11b). In both cases, the $m_x - h_z$ and $m_y - h_z$ loops lie almost along h_z axis, resulting Q_x and Q_y equal to zero respectively. Thus a dynamic transition occurs (as the temperature decreases) at a certain temperature from a symmetric ($Q_z = 0$; $\vec{Q} = 0$) to an asymmetric ($Q_z \neq 0$; $\vec{Q} \neq 0$) dynamic phase in presence of an *axial* magnetic field.

What was observed in the case of *off-axial* transition ? Recently studied [31] off-axial transition

shows similar dynamic transition via breaking the symmetry of $m_z - h_x$ loop in presence of an off-axial field (along perpendicular to the anisotropy direction i.e., x-direction). Here, at high temperature ($T = 1.8$) the $m_z - h_x$ loop is symmetric (and $Q_z = 0$) and $m_x - h_x$ loop is also symmetric ($Q_x = 0$) (fig.11c). At some lower temperature ($T = 0.6$), the $m_z - h_x$ loop becomes asymmetric ($Q_z = 0$) and $m_x - h_x$ loop remains still symmetric ($Q_x = 0$) (fig.11d). In both temperatures $Q_y = 0$. So, here also a dynamic transition occurs (as the temperature decreases) at a certain temperature from a symmetric ($Q_z = 0$; $\vec{Q} = 0$) to an asymmetric ($Q_z \neq 0$; $\vec{Q} \neq 0$) dynamic phase in presence of an *off-axial* magnetic field. Interestingly, it may be noted here that in higher temperature the $m_z - h_x$ loop is 'marginally symmetric' (lies very close to h_x axis) rather than a symmetric loop (symmetrically distributed away from and about h_x line). Strictly speaking, the dynamic transition occurs here from a 'marginally symmetric' (loop does not widen up) to an asymmetric phase. One can differenciate the symmetric phase from the 'marginally symmetric' phase by considering the loop area of that loop whose symmetry breaking is considered in the transition. In the symmetric phase loop is sufficiently widen up resulting nonzero loop area. In Fig.11a, the $m_z - h_z$ loop area is 0.686 (symmetric loop; $Q_z = 0$). But the 'marginally symmetric' loops ($m_z - h_x$) have vanishingly small area (0.01)(see Fig.11c) and $Q_z = 0$. It may be noted that, in the case of off-axial transition, if the magnetic field applied along the x-direction only (oscillating sinusoidally in time) the $m_x - h_x$ loop is always symmetric (consequently $Q_x = 0$) irrespective of the value of temperature and the strength of anisotropy D (z-axis). Similarly, for any field applied along y-direction only, the $m_y - h_y$ loop is found to be always symmetric (i.e., $Q_y = 0$) irrespective of value of T and D. But in both cases, whether the off-axial loops i.e., $m_z - h_x$ or $m_z - h_y$ will be symmetric (rather 'marginally symmetric') or asymmetric that depends upon the values of temperature T, anisotropy D and the magnetic field amplitude h_x^0 (or h_y^0). These results signify that without anisotropy the dynamic transition (associated to the dynamic symmetry breaking) cannot be observed in the classical Heisenberg model.

To investigate the dependence of transition temperature on the strength of anisotropy (D) in the case of axial transition, the temperature variation of dynamic order parameter component Q_z was studied for different values of D. Figure 12 shows the temperature variation of Q_z for different values of D. Here, like the case of off-axial transition [31] the transition temperature increases as the strength of anisotropy increases. It is observed that the axial transition is discontinuous for lower values of anisotropy strength D (i.e., 0.5, 2.5 etc.) and it becomes continuous for higher values of D (i.e., 5.0 15.0 etc.). In the Ising limit ($D \to \infty$) the axial transition is also shown in the same figure for $D = 400$). This choice of the value of $D(= 400)$ is not arbitrary. In the case of equilibrium transition it was checked by MC simulation that the value of the magnetisation at any temperature (in the ferromagnetic region) becomes very close to that (at that temperature) obtained in the Ising model if the strength of anisotropy is chosen above 300.

The temperature variations of dynamic order parameter component Q_z in the case of off-axial transition was also studied and shown in figure 13 for different values of D. This shows that the transition temperature increases as D increases. Here, the transition is continuous for all values of strength of anisotropy D. The transition for $D = 400$ ($D \to \infty$ limit) was compared with that in the case of Ising model. This shows that both are continuous and occur at the same point ($T \approx 4.5$) which is very close to the Monte Carlo results of equilibrium ferro-para transition temperature ($T_c \simeq 4.511$) [25] in 3-dimensional Ising model.

The nonequilibrium dynamical phase transition in the uniaxially anisotropic Heisenberg model, in presence of magnetic field oscillating sinusoidally in time, is studied by Monte Carlo simulation. Two cases were studied in this paper. (i) magnetic field oscillating sinusoidally in time is applied only along the direction of anisotropy, (ii) magnetic field applied only along the direction perpendicular to the direction of anisotropy. The transition observed in the first case is named axial and that corresponding to the second case is called off-axial. A comparative study between axial and off-axial transition is reported in this paper. Three important aspects are considered here. (a) symmetry breaking, (b) the order of the transition and (c) the transition in infinite anisotropic limit.

A dynamic symmetry breaking is observed with this dynamic transition. In the case of axial transition the dynamic transition occured as the temperature decreases from a symmetric to an asymmetric phase. Whereas, in off-axial case this symmetry breaking takes place from a 'marginally symmetric' to an asymmetric phase. The reason behind it is as follows: in the case of axial transition by the application of axial field (oscillating sinusoidally in time) there is a chance that the spin component along the z-direction may be reversed in opposit direction which would lead to sufficiently wide and symmetric $m_z - h_z$ loop. But in the case of off-axial transition it is not possible to reverse the z-component of spin by applying a field (oscillating sinusoidally in time) perpendicular to the direction of uniaxial anisotropy. In this case the value of the z-component of magnetisation m_z is almost zero. As a result the $m_z - h_x$ loop lies on $h_x = 0$ axis and hence the loop is marginally symmetric.

In both the cases (axial and off-axial) the transition temperature increases as the strength of anisotropy increases provided the amplitude of the applied field remains same. The strength of anisotropy tries to align the spin vector along the direction of anisotropy. So, as the strength increases it becomes harder to break the symmetry and consequently more thermal fluctuation is required to break the symmetry. As a result, the transition temperature increases as the strength of anisotropy increases. But the difference is the nature of transition. In the case of axial field the transition is discontinuous for lower values of anisotropy and it becomes continuous for higher values of anisotropy. The reason behind it is, the axial transition occurs in presence of axial field which reverses the z-component of magnetisation. So, in lower values of anisotropy the spin vector becomes comparatively more flexible and the transition occurs mechanically in presence of axial field at lower temperature and it is discontinuous. As the anisotropy increases the effect of axial field (of same value) becomes weak and the transition is driven by thermal fluctuations and the transition is continuous. In the case of off-axial transition, the off-axial field cannot reverse the z-component of magnetisation. But as the value of off-axial field increases, the value of x-component of magnetisation increases at the cost of z-component of magnetisation. The transition is driven by thermal fluctuations and continuous.

What will be the situation in the limit of infinite strength of anisotropy ? In the case of axial transition it was observed that the transition temperature for infinitely anisotropic Heisenberg model differs from that obtained in an Ising model. Although the equilibrium transitions in infinitely anisotropic Heisenberg model and that in the Ising model gives the same transition temperature, the nonequilibrium transition temperatures in those two cases are not same. Since the magnetic field applied in z-direction oscillating sinusoidally in time, keeps the system always away from the equilibrium, the system does not become an Ising system even in infinite anisotropy limit. As a result, the dynamic transition temperature in infinitely anisotropic Heisenberg model cannot be same for that obtained in the Ising model. But in the case of off-axial transition, the transition temperatures in infinitely anisotropic Heisenberg model and that in the Ising model becomes exactly equal. The reason behind it is as follows: in the case of off-axial transition the field is applied perpendicular to the direction of anisotropy. The effect of axial field oscillating sinusoidally in time has no effect in infinite anisotropic limit. Though the magnetic field applied in the x-direction oscillating sinusoidally in time, the infinite anisotropic Heisenberg model becomes an Ising model in statistical and thermal equilibrium. Hence, the infinitely anisotropic Heisenberg model in presence of off-axial field maps into the Ising model in zero field. That is why the nonequilibrium transition in infinitely anisotropic Heisenberg model in presence of off-axial field and the Ising model (in zero external field) give the same result.

One important point may be noted here regarding the dynamics chosen in this simulation. Since, the spin component does not commute with the Heisenberg Hamiltonian the spin component has an intrinsic dynamics. Considering this intrinsic dynamics there was a study [22] about structure factor and transport properties in XY- model. However, in this paper, the motivation is to study the nonequilibrium phase transition driven by thermal fluctuations. To study this, one should choose the dynamics which arises due to the interaction with thermal bath. Since the objective is different, in

this paper, the dynamics chosen here (which arises solely due to the interaction with thermal bath), is Metropolis dynamics. The effect of intrinsic spin dynamics is not taken into account.

(v) Multiple dynamic transition

In this case the MC study was done [32] for a polarised magnetic field having the form: $\vec{h} = ih_x + jh_y + kh_z = ih_{0x}\cos(\omega t) + kh_{0z}\sin(\omega t)$. One can readily check that $h_x = h_{0x}\cos(\omega t)$ and $h_z = h_{0z}\sin(\omega t)$ yield, after the elimination of time

$$\frac{h_x^2}{h_{0x}^2} + \frac{h_z^2}{h_{0z}^2} = 1 \tag{21}$$

The simulational study is done for a simple cubic lattice of linear size $L = 20$. The total (vector) dynamic order parameter can be expressed as $\vec{Q} = iQ_x + jQ_y + kQ_z$. The instantaneous energy $e(t) = -J\sum_{<ij>}\vec{S}_i \cdot \vec{S}_j - D\sum_i (S_i^z)^2 - \vec{h}\cdot\sum_i \vec{S}_i$ is also calculated. The time averaged instantaneous energy is $E = \frac{1}{\tau}\oint e(t)dt$. The rate of change of E with respect to the temperature T is defined as dynamic specific heat $C(= \frac{dE}{dT})$ [7]. The dynamic specific heat C is calculated from energy E, just by calculating the derivative using the three-point central difference formula, given below.

$$C = \frac{dE}{dT} = \frac{E(T+\delta T) - E(T-\delta T)}{2\delta T} \tag{22}$$

For the elliptically polarised (equation 21) magnetic field, where the resultant field lies in X-Z plane, the amplitudes of fields are taken as $h_{0x} = 0.3$ and $h_{0z} = 1.0$ and the frequency $f = 0.02$. The strength of uniaxial anisotropy is taken $D = 0.2$. This value of D is obtained by rigorous searching to have these interesting results and kept constant throughout the study. However, there must be variations in transition points depending on the values of D. It is observed that for Higher values of D the multiple transition phenomenon disappears. The values of field amplitudes and frequency are also obtained by searching.

The temperature variations of the dynamic order parameter components (Q_x, Q_y, Q_z) are studied and the results are depicted in Fig.14(a). As the system is cooled down, from a high temperature disordered ($\vec{Q} = 0$) phase, it was observed that, first the system undergoes a transition from dynamically disordered ($\vec{Q} = 0$) to a dynamically *Y-ordered* (only $Q_y \neq 0$) phase. This may be called as the first phase (P_1) and the transition temperature is T_{c1}. This phase can be characterised as P_1: ($Q_x = 0$, $Q_y \neq 0$, $Q_z = 0$). Here, the resultant vector of elliptically polarised magnetic field lies in x-z plane and the dynamic ordering occurs along y-direction. So, this is clearly an off-axial transition [31]. In the case of this type of off-axial transition the dynamical symmetry (in any direction; y-direction here) is broken by the application of the magnetic field in the perpendicular direction (lies in the x-z plane here). As the system cools down, it retains this particular dynamically ordered phase (P_1) over a considerable range of temperatures. As the temperature decreases further, a second transion was observed. Here, the system becomes dynamically ordered both in X- and Z-directions at the cost of Y-ordering. In this new dynamic phase,P_2: ($Q_x \neq 0$, $Q_y = 0$, $Q_z \neq 0$). In this phase the dynamical ordering is planar (lies on x-z plane). The ordering occurs in the same plane on which the field vector lies. This transition is axial [31]. This phase may be called the second phase (P_2) and the transition (from first phase to the second phase) temperature is T_{c2}. As the temperature decreases further, the X- and Z-ordering increases. At some lower temperature, a third transition was observed, from where the X-ordering starts to decrease and only Z-ordering starts to increase quite rapidly. This third phase can be designated as P_3: ($Q_x \neq 0$, $Q_y = 0$, $Q_z \neq 0$). Although the characterisation of P_2 and P_3, in terms of the values of dynamic order parameter components, looks similar there exists an important difference between these two phases. In the phase P_2, both Q_x and Q_z increases as the tempereture decreases but in the phase P_3, Q_x decreases as the temperature decreases (see Fig.14(a)). So these two phases P_2 and P_3 distinctly differs from each other. In this phase the dynamical ordering is also axial (along Z-axis or anisotropy axis). The system continues to

increase the dynamical Z-ordering as the temperature decreases further. The low temperature phase is only dynamically Z-ordered. That means the systems orders dynamically (only $Q_z \neq 0$) along the Z-direction (direction of anisotropy) only at very low temperatures. Zero temperature dynamic phase (for such polarised field) can be characterised as $Q_x = 0$, $Q_y = 0$ and $Q_z = 1.0$.

To detect the dynamic transitions and to find the transition temperatures the temperature variation of the energy E is plotted in Fig.14(b). From this figure it is clear that there are three dynamic transitions occur in this case. The transition points are the inflection points in $E - T$ curve. The temperature derivative of the energy E is the dynamic specific heat C. The temperature variation of C is shown in Fig.14(c). The three dynamic transitions are very clearly shown by three peaks of the specific heat plotted against the temperature T. From this figure the transition temperatures are calculated (from the peak positions of $C - T$ curve). First transition (right peak) occurs around $T_{c1} = 1.22$, the second transition (middle peak) occurs at $T_{c2} = 0.94$ and the third (left peak) transition occurs around $T_{c3} = 0.86$.

This study was further extended for other values of h_{0x} keeping other parameters fixed. It was found that this three transitions senario disappears for higher values of h_{0x}. For example, for $h_{0x} = 0.9$, the second phase P_2 disappears. In this case, the $C - T$ curve shows two peaks. It was also observed that for $h_{0x} = 0.2$, $h_{0z} = 0.2$ (keeping all other parameter fixed) the system shows single transition and only dynamically orders along Z-direction.

In the present study, the external time dependent magnetic field was taken elliptically polarised where the resultant field vector rotates on X-Z plane. For the lower values of anisotropy and a specific range of the values of field amplitudes the system undergoes multiple dynamic phase transitions. Here, three distinct phases are identified. In this paper, this observation is just briefly reported. This multiple dynamic phase transition in anisotropic Heisenberg ferromagnet in presence of elliptically polarised field, is observed here by Monte Carlo simulation. An alternative method, to check this phenomenon, may be to use Landau-Lifshitz-Gilbert equation of motion [30] with Langevin dynamics. Another important thing should be mentioned here regarding the possible explanation of multiple dynamic phase transitions (axial and off-axial transitions) observed in the anisotropic Heisenberg model. One possible reason may be the coherrent rotation of spins. Where the dynamic phase transition in the Ising model can be explained simply by spin reversal and nucleation [21]. But to know the responsible mechanism behind the multiple dynamic phase transitions, observed in anisotropic Heisenberg ferromagnet in presence of polarised magnetic field, details investigations are required.

The variations of the dynamic phase boundaries with frequency and the strength of anisotropy is quite interesting to be studied. This study also indicates that the system will show a very rich phase diagram with multicritical behaviour. The finite size analysis is also necessary in order to distinguish the crossover effects from the true phase transitions. This requires huge computational task which will take much time. This work is in progress [33] and the details will be reported later.

In the context of multiple dyanmic transition in anisotropic Heisenberg ferromagnet it should be mentioned here that a recent study [29] of anisotropic Heisenberg thin ferromagnetic film shows a double dynamic phase transition for surface and bulk order parameter. The Hamiltonian for the classical Heisenberg ferromagnet with a bilinear exchange anisotropy λ, in presence of competing surface fields as well as pulsed oscillatory fields, was taken [29] as

$$H = -J\Sigma_{<ij>}[(1-\lambda)(S_i^x S_j^x + S_i^y S_j^y) + S_i^z S_j^z)] - \Sigma_{i\epsilon surface1}\vec{H}_1 \cdot \vec{S}_i - \Sigma_{i\epsilon surfaceD}\vec{H}_D \cdot \vec{S}_i - H(T)\Sigma_i S_i^z \quad (23)$$

where $\vec{H}_1$ and $\vec{H}_D$ are the static applied surface fields and the time dependent field $H(t)$ was taken to have a pulsed form with

$$H(t) = \begin{cases} -H_0, \frac{2(k-1)\pi}{\omega} < t \leq \frac{(2k-1)\pi}{\omega} \\ H_0, \frac{2(k-1)\pi}{\omega} < t \leq \frac{2k\pi}{\omega} \end{cases} \quad (24)$$

Where h_0 is the amplitude and ω is the angular frequency of the oscillatory external field and k is an integer ($k = 1, 2, 3....$) representing the number of periods of the pulsed oscillatory external field.

The model film was taken [29] a simple lattice of size $L \times L \times D$. D is the thickness of the film of planar dimension $L \times L$. The system is subject to competing applied surface fields in layers $n = 1$ and $n = D$ of the film with $\vec{H}_1 = h\hat{z}\delta_{i1}$ and $\vec{H}_D = -h\hat{z}\delta iD$. The Monte Carlo study was done [29] using Metropolis algorithm for $D = 12$ and $L = 32$. They calculated the surface and bulk order parameter $< Q^{surface} >$ and $< Q^{bulk} >$ respectively and studied as a function of temperature. From Fig. 15 it is clear that the critical temperature for the DPT in the surface layers is not the same as that for bulk of the film. A double DPT was observed [29] for a anisotropic Heisenberg film for competing surface fields and pulsed oscillatory fields.

The off-axial[31] and multiple DPT[32, 29] in anisotropic Heisenberg ferromagnet shows quite interesting DPT which was not observed in Ising model [1] and a rich phase diagram is expected here [33].

5. Experimental evidences of dynamic phase transitions:

Several experimental works [34, 35, 36] were performed to investigate the hysteretic responses as well as the dynamic transitions in ferromagnetic samples. In a recent experiment, Jiang et al [36] studied the frequency-dependent hysteresis of epitaxially grown ultrathin (2 to 6 monolayer thick) Co films on a Cu(001) surface at room temperature. The films have strong uniaxial magnetisation with two ferromagnetic phases of opposite spin orientations. This magnetic anisotropy makes it appropriate to represent the system by an Ising-like model. The external magnetic field $h(t)$ on the system was driven sinusoidally in the frequency ($f = \omega/2\pi$) range 0.1 to 500 Hz and in the amplitude (h_0) range 1 to 180 Oe. Here of course the time-varying current or the magnetic field induces an eddy current in the core, which results in a counter-field reducing the effective magnitude of the applied field. The surface magneto-optical Kerr effect technique was used to measure the response magnetisation $m(t)$. A typical variation of the loop area A with the driving frequency f, at room temperature and at fixed external field amplitude h_0, is shown in Fig. 16(a). Fig. 16(b) shows clearly a signature of dynamic transition and corresponding symmetry breaking. Here, the dynamic order parameter Q is plotted against the field amplitude h_0 at a fixed frequency and emperature. The inset of Fig. 16(b) shows that for lower values of field amplitude i.e., 12.0 Oe (left inset) the phase is dynamically ordered ($Q \neq 0$) and asymmetric, and for higher values of field amplitude i.e., 48.1 Oe, the $m - h$ loop is symmetric and the phase is dynamically disordered ($Q = 0$). This experimental observation supports the theoretical results of dynamic transition and dynamical symmetry breaking. However, experimental study of the entire phase boundary is yet to be done.

6. Summary:

The dynamical phase transitions in model ferromagnetic systems (Ising and Heisenberg) in the presence of sinusoidally oscillating magnetic field is reviewed here. This nonequilibrium dynamical phase transition is a prototype of nonequilibrium phase transition. The kind of nonequilibrium phase transition discussed above is observed very recenly. Here, only the observations are reviewed. The detailed mechanism behind this type of nonequilibrium transition is not yet known clearly. In Ising models, the mechanism was tried to understood in view of nucleation [21]. However, in Heisenberg model the axial, off-axial [31] and very recently observed multiple dynamic transition [32] are just observed and the mechanisms responsible for those transition are not yet known. The coherrent spin rotation [30] may be a possible reason for this. The experimental observations of the dynamic phase transition [36] are only made for Ising like (highly anisotropic) systems. The experimental studies are required to observe the special dynamic transitions in Heisenberg model.

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Figure Captions

Figure 1: Schematic time variation of the response magnetisation $m(t)$ compared to that of the oscillating field $h(t)$ for different values of frequency ω and amplitude h_0 of the oscillating field and temperature T of the system. The results are in fact actual Monte Carlo simulation results for an Ising model on a square lattice with the values for h_0 and T as indicated in the Figures. The Figures on the right hand side show the corresponding $m-h$ loops. The values for loop area $A = \oint m dh$ and the dynamic order parameter Q are also indicated in these figures. As one can see, the first two cases correspond to $Q = 0$, while the other two correspond to dynamically broken symmetric phase (with $Q \neq 0$). The first figure and the last correspond to $A \simeq 0$, while the middle two correspond to nonvanishing A.

Figure 2: Phase diagrams in the h_0-T plane for various values of ω gives the functional form of the transition temperature $T_d(h_0, \omega)$ for the dynamic phase transition: Monte Carlo results **(a)** for system sizes $L = 100$ in $d = 2$, and **(b)** for $L = 20$ in $d = 3$. Below $T_d(h_0, \omega)$, Q acquires a nonzero value in F phase and $Q = 0$ in P phase. Different symbols denote different phase boundary lines corresponding to different frequencies (ω): ($\Box$) $\omega = 0.418$, ($\triangle$) $\omega = 0.208$, ($\diamond$) $\omega = 0.104$ in **(a)**; and ($\diamond$) $\omega = 0.418$, ($\Box$) $\omega = 0.202$, ($\circ$) $\omega = 0.104$ in **(b)**. The locations of the tricritical points (TCP) are indicated by the circle. The insets show the nature of the transition just above (I: $h_0 = 2.2$ and 4.4 in **(a)** and **(b)** respectively) and below (II: $h_0 = 1.8$ and 3.6 in **(a)** and **(b)** respectively) the tricritical points along the phase boundaries.

Figure 3: Monte Carlo results of the temperature (T) variation of 'relaxation' time (Γ) for two different values of field amplitudes (h_0):the bullet represents $h_0 = 1.0$ and the diamond represents $h_0 = 0.5$. Solid lines show the temperature (T) variations of dynamic order parameter Q. Inset shows a typical 'relaxation' of Q plotted against the number of cycles (n). The solid line is the best fit exponential form of the data obtained from MC simulation. Here, $L = 100$, $\omega = 2\pi \times 0.04$.

Figure 4: Monte Carlo results of the temperature variations of C_{tot} for two different values of h_0: the filled square represents $h_0 = 0.8$ and the filled triangle represents $h_0 = 0.4$. Solid lines represent the temperature variations of Q. Inset shows the temperature variations of E_{tot} for two different values of h_0:(I) $h_0 = 0.8$ and (II) $h_0 = 0.4$. Here, $L = 100$, $\omega = 2\pi \times 0.01$.

Figure 5: $L^2 \mathrm{Var}|Q|$ vs dimensionless frequency, $1/R$. The "disordered phase," ($\langle |Q| \rangle$=0), lies on the low-frequency side of the peaks. The "ordered phase", ($\langle |Q| \rangle \neq 0$), lies on the high-frequency side. Lines connecting data points are guides to the eye. The statistical error bars are estimated by partitioning the data into ten blocks. Error bars smaller than the symbol sizes are not shown. [After S. W. Sides et al, *Phys. Rev. Lett.*, **81** (1998) 834.]

Figure 6: (a) The histograms of the normalized distributions of the dynamic order parameter Q for different temperatures ($T = 0.20J/K_B$, $0.28J/K_B$, $0.30J/K_B$ and $0.40J/K_B$) and for the fixed value of field amplitude h_0. All the figures are plotted in the same scales. (b) The normalized distributions of the dynamic order parameter Q (in the 2nd order and close to the transition region) for three different temperatures ($T = 1.48J/K_B$, $1.50J/K_B$, $1.55J/K_B$) and fixed field amplitude $h_0 = 0.3J$.

Figure 7: The histogram of normalized ($\int P_r(\tau_r) d\tau_r = 1$) distribution ($P_r(\tau_r)$) of the residence time (τ_r). The dotted line is the exponential best fit of the envelope of the distribution.

Figure 8: Histograms representing $P(Q)$ (A) in a small system (L=16 with H=2.0J and $t_{1/2}$=50 MCSS) for different temperatures. (B) in a large system (L=180 with H=2.0J and $t_{1/2}$=50 MCSS) for different temperatures. [After G. Korniss et al Phys. Rev. E 66 (2002) 056127]

Figure 9: Dynamic transition due to randomly varing fields in time. **(a, b)** Typical time variation of magnetisation $m(t)$ compared to that of the stochastically varying field $h(t)$ in a Monte Carlo study in $d = 2$, with $L = 100, T = 1.7$: $h_0 = 1.0$ for **(a)** and $h_0 = 3.0$ for **(b)**. **(c)** The corresponding dynamic transition phase boundary (separating the regions with average magnetisations Q zero from nonzero) in the field width (h_0) - temperature (T) plane. The data points are obtained using both sequential updating ($\diamond$) and random updating ($\bullet$) in the Monte Carlo simulation.

Figure 10: (a) The phase boundary (of the dynamic transition) in $w - h_0$ plane. The tricritical point (TCP) lies within the encircled region. The boundary of the circle is the uncertainty associated in locating the TCP, (b) two typical transitions just below and above the tricritical point which show the different natures (discontinuous/continuous) of the transitions.

Figure 11: Symmetry breaking in axial and off-axial transitions. The plot of instantaneous magnetization components against the instantaneous field components. (a) $m_x(t) - h_z(t)$ and $m_z(t) - h_z(t)$ loops for $D = 2.5$, $h_z^0 = 0.5$ and $T = 2.2$, (b) $m_x(t) - h_z(t)$ and $m_z(t) - h_z(t)$ loops for $D = 2.5$, $h_z^0 = 0.5$ and $T = 1.0$, (c) $m_x(t) - h_x(t)$ and $m_z(t) - h_x(t)$ loops for $D = 0.5$, $h_x^0 = 0.5$ and $T = 1.8$ and (d) $m_x(t) - h_x(t)$ and $m_z(t) - h_x(t)$ loops for $D = 0.5$, $h_x^0 = 0.5$ and $T = 0.6$.

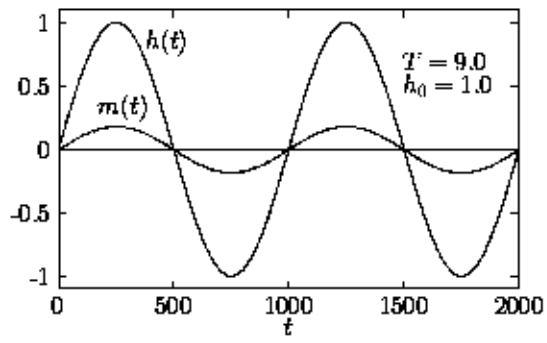
Figure 12: The axial dynamic transitions. Temperature (T) variations of dynamic order parameter components Q_z for different values of anisotropy strength (D) represented by different symbols. $D = 0.5(\diamond)$, $D = 2.5(+)$, $D = 5.0(\square)$, $D = 15.0(\times)$ and $D = 400.0(\triangle)$. In all these cases for the *axial* transitions $h_z^0 = 0.5$. The data for the temperature variation of dynamic order parameter in the Ising model (for $h_z^0 = 0.5$ and $f = 0.001$) are represented by $\star$. Continuous lines in all cases are just connecting the data points.

Figure 13: The off-axial dynamic transitions. Temperature (T) variations of dynamic order parameter components Q_z for different values of anisotropy strength (D) represented by different symbols. $D = 0.5(\diamond)$, $D = 2.5(+)$, $D = 5.0(\square)$, $D = 15.0(\times)$ and $D = 400.0(\triangle)$. In all these cases for *off − axial* transitions $h_x^0 = 0.5$. The data for the zero-field ferro-para equilibrium Ising transition are represented by $\star$. Continuous lines in all cases are just connecting the data points.

Figure 14: (a) The temperature variations of the components of dynamic order parameters. Different symbols represent different components. Q_x (diamond), Q_y (circle) and Q_z (bullet). This diagram is for $D = 0.2$ and for elliptically polarised field where $h_{0x} = 0.3$ and $h_{0z} = 1.0$. The size of the errorbars of Q_x, Q_y and Q_z close to the transition points is of the order of 0.02 and that at low temperature (e.g., below $T = 0.5$) is around 0.003. (b) The temperature variation of the dynamic energy (E) for $D = 0.2$, $h_{0x} = 0.3$ and $h_{0z} = 1.0$. The vertical arrows represent the transition points. (c) The temperature variation of dynamic specific heat ($C = \frac{dE}{dT}$) for $D = 0.2$, $h_{0x} = 0.3$ and $h_{0z} = 1.0$. Vertical arrows show the peaks and the transition points.

Figure 15: Surface order parameter $< Q^{surface} >$ and bulk order parameter $< Q^{bulk} >$ for the film, plotted as a function of temperature (T^*) for the value of pulsed oscillatory field $H_0 = 0.3$.
[After Jang et al, Phys. Rev. B 67 (2003) 094411]

Figure 16: Experimental results for the dynamic hysteresis loop area A and the dynamic order parameter Q [36]. **(a)** The results for the loop area A as a function of frequency f is plotted at a fixed ac current of 0.4 Amp. The direction of the magnetic field is parallel to the film plane. The insets show plots of $m - h$ loops for the following particular values of the field amplitudes h_0: (i) $h_0 = 48.0$ Oe (top inset) and (ii) $h_0 = 63.0$ Oe (bottom inset). **(b)** The dynamic order parameter Q, i.e, the average magnetisation over a cycle, is plotted against the field amplitude at a fixed frequency $f = 4$ Hz. The insets show plots of $m - h$ loops for the following particular values of the field amplitudes h_0: (i) $h_0 = 48.1$ Oe (right inset) and (ii) $h_0 = 12.0$ Oe (left inset). [After Q. Jiang et al. Phys. Rev. B. 52 (1995) 14911]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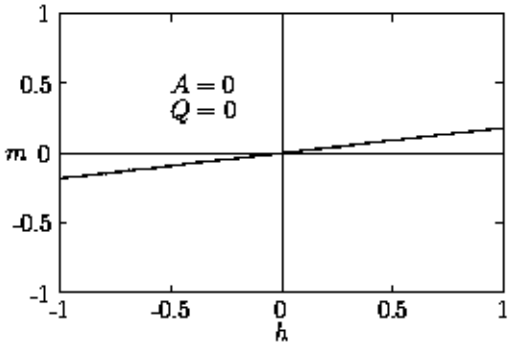
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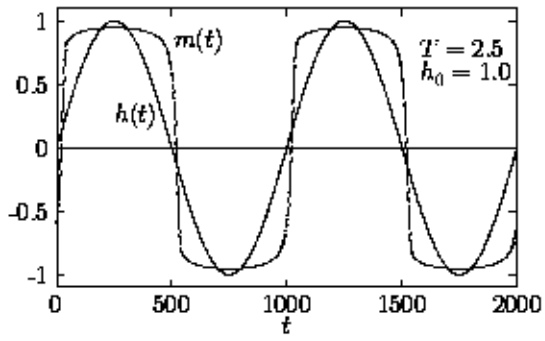
h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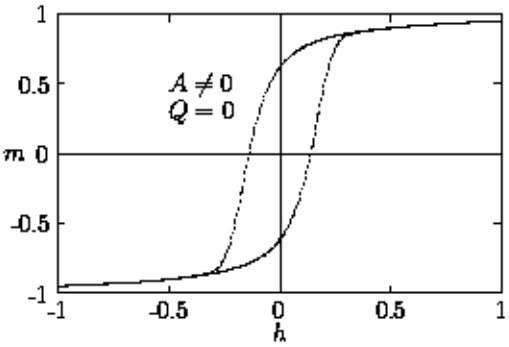
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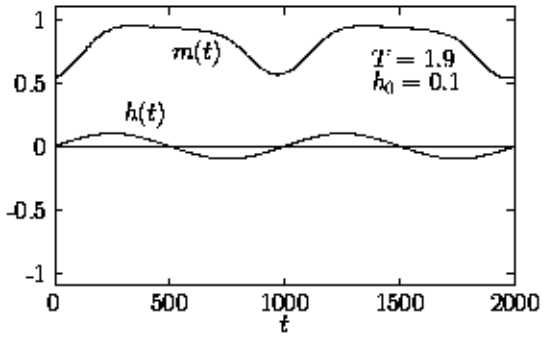
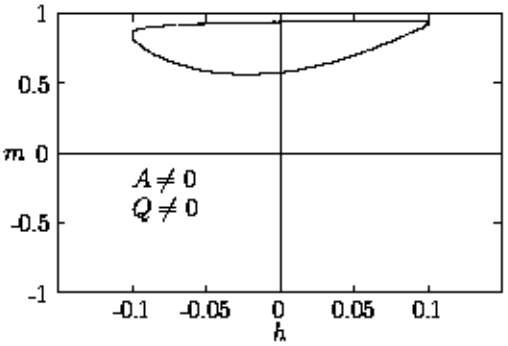
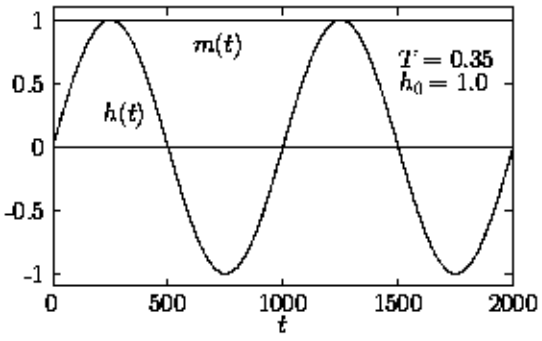
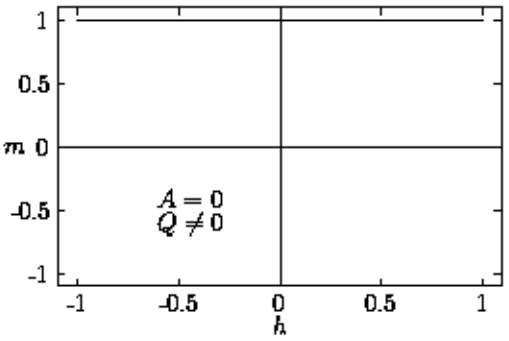
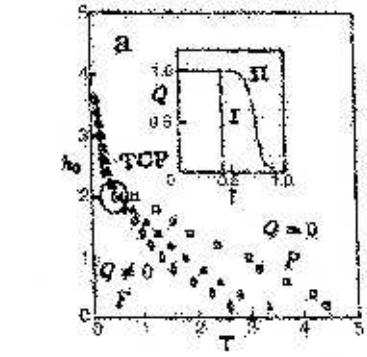
Q = 0
m
h

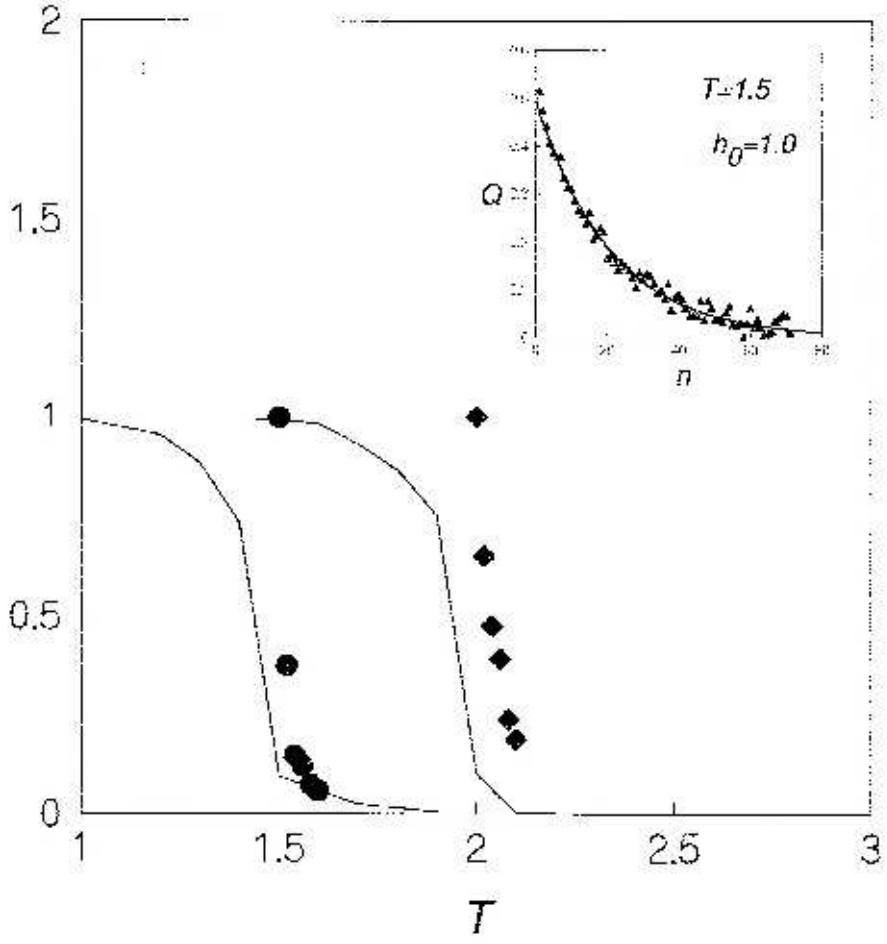

m(t)
h(t)
T = 1.9
h0 = 0.1
t

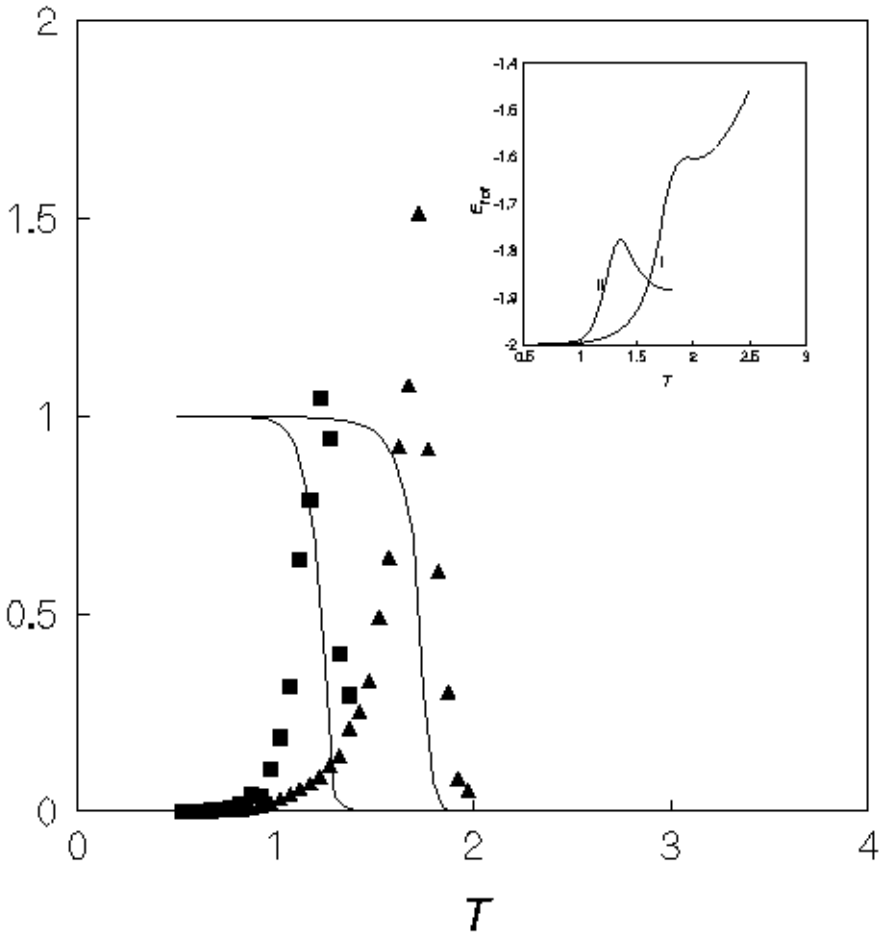

A ≠ 0
Q ≠ 0
m
h

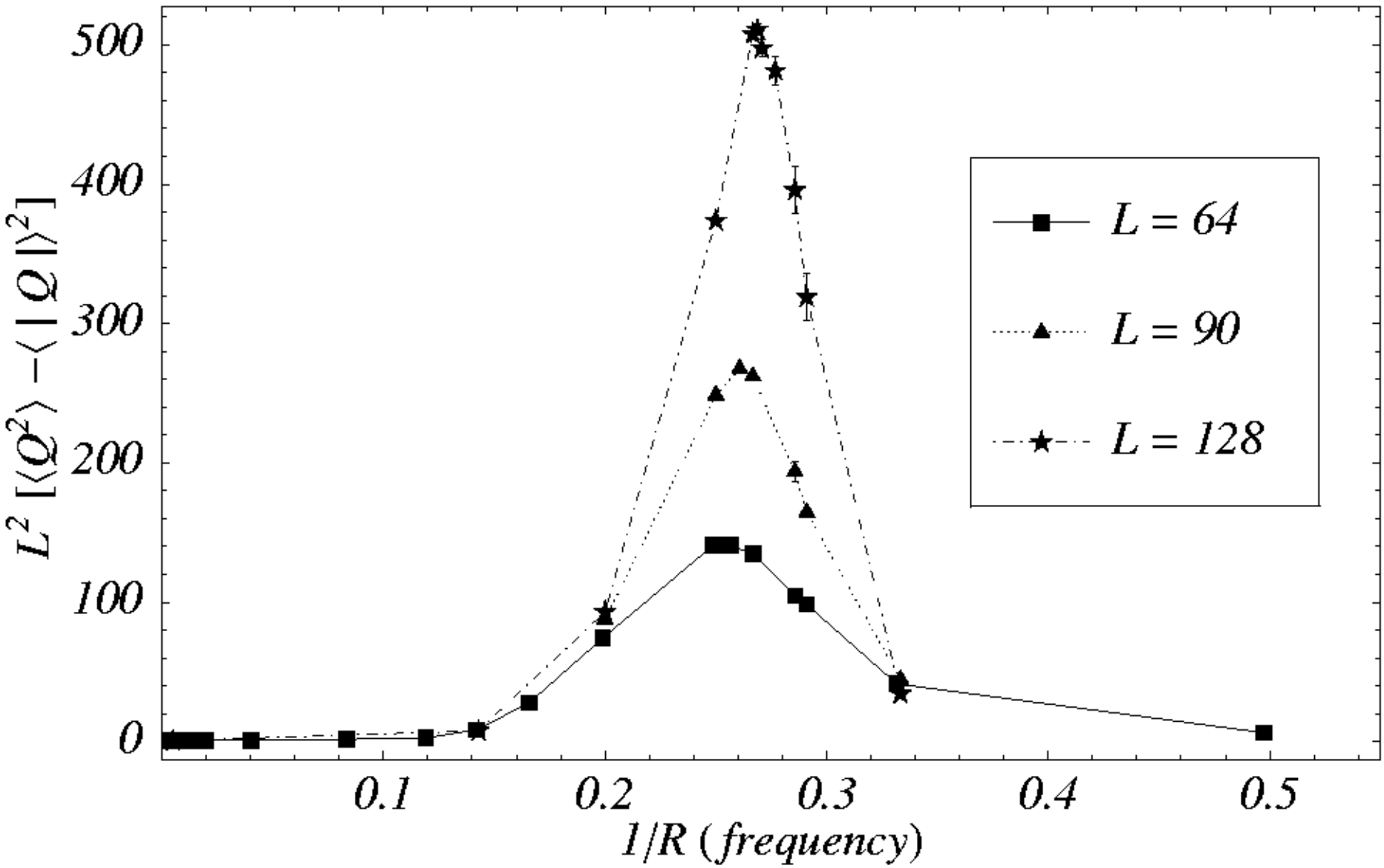

m(t)
h(t)
T = 0.35
h0 = 1.0
t

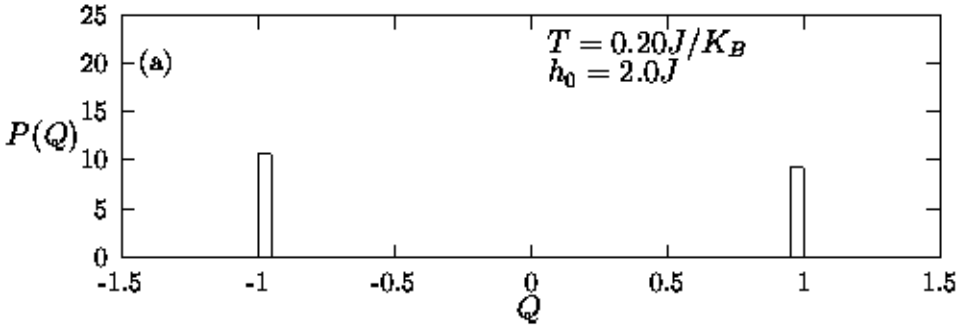

A = 0
Q ≠ 0
m
h

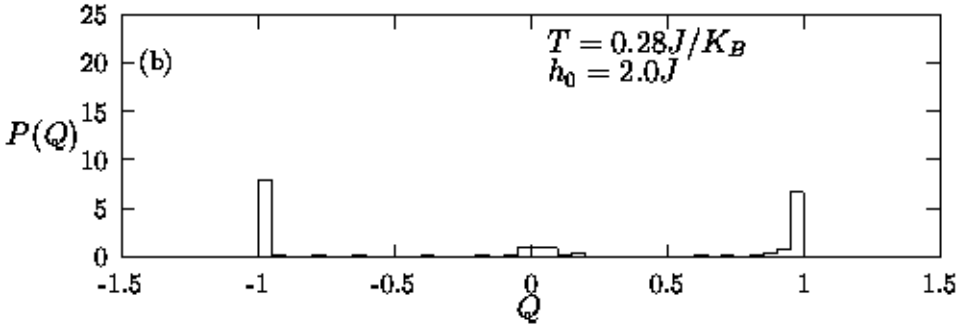

a
Q
I
II
TCP
h_0
$Q = 0$
$Q \neq 0$
F
P
T
b
Q
I
II
TCP
h_0
4.00
2.00
$Q = 0$
$Q \neq 0$
F
P
T

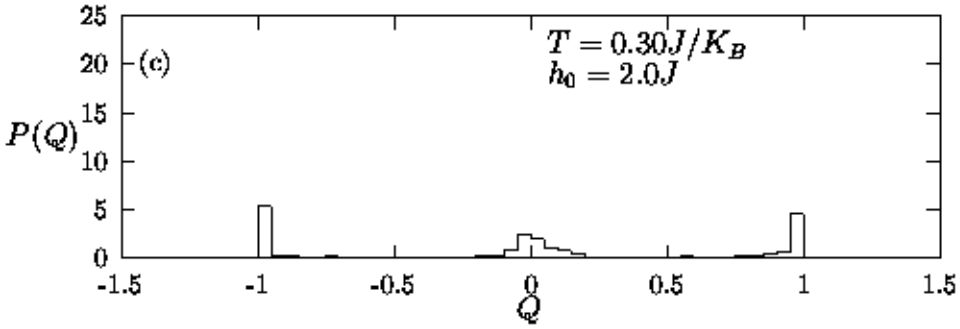

2
1.5
1
0.5
0
1
1.5
2
2.5
3
T
T=1.5
h0=1.0
Q
n

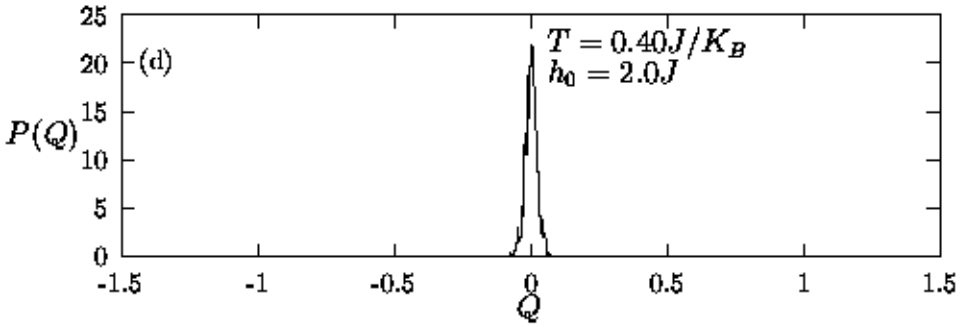

2
1.5
1
0.5
0
0
1
2
3
4
T
-1.4
-1.5
-1.6
-1.7
-1.8
-1.9
-2
0.5
1
1.5
2
2.5
T
I
II

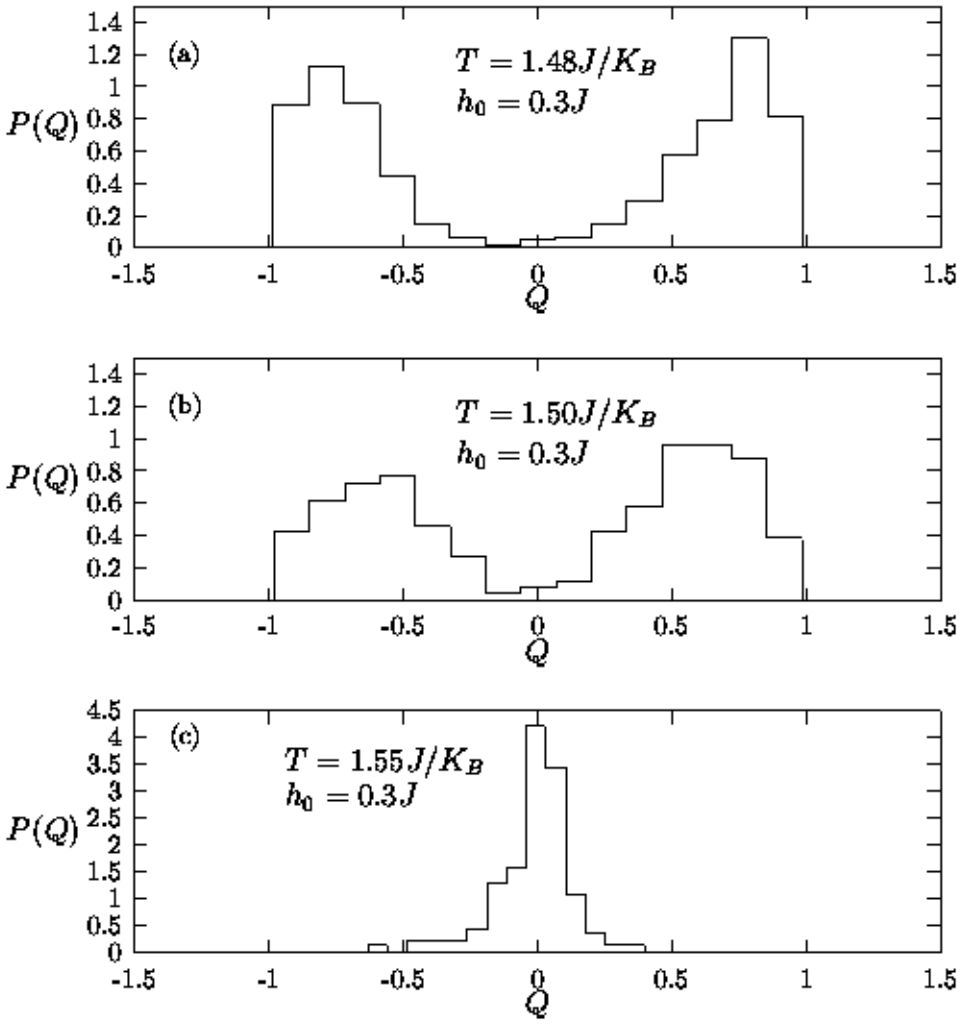

500
400
300
200
100
0
$L^2\ [\langle Q^2\rangle - \langle\,|\,Q\,|\,\rangle^2]$
0.1
0.2
0.3
0.4
0.5
1/R (frequency)
L = 64
L = 90
L = 128

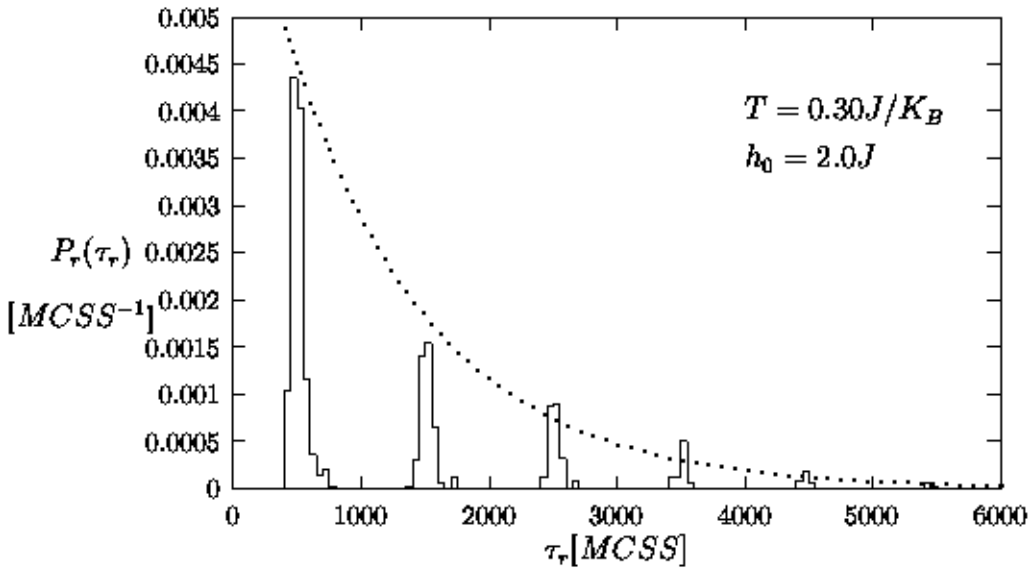

(a)
$T = 0.20J/K_B$
$h_0 = 2.0J$
$P(Q)$
Q

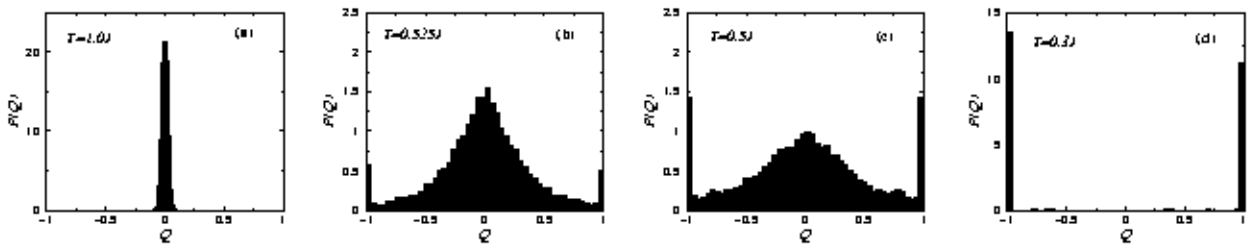

(b)
$T = 0.28J/K_B$
$h_0 = 2.0J$
$P(Q)$
Q

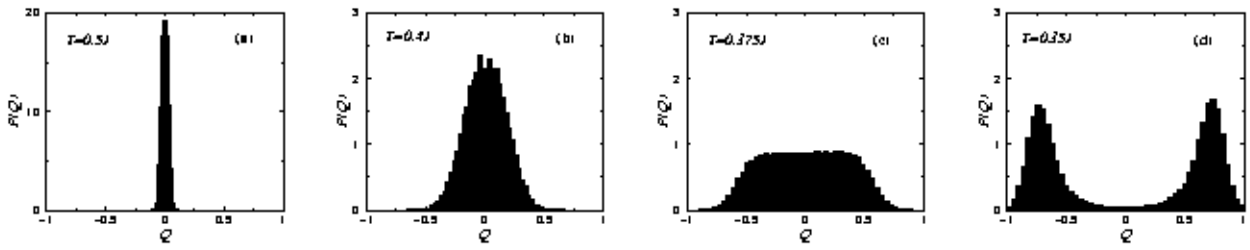

(c)
$T = 0.30J/K_B$
$h_0 = 2.0J$
$P(Q)$
Q

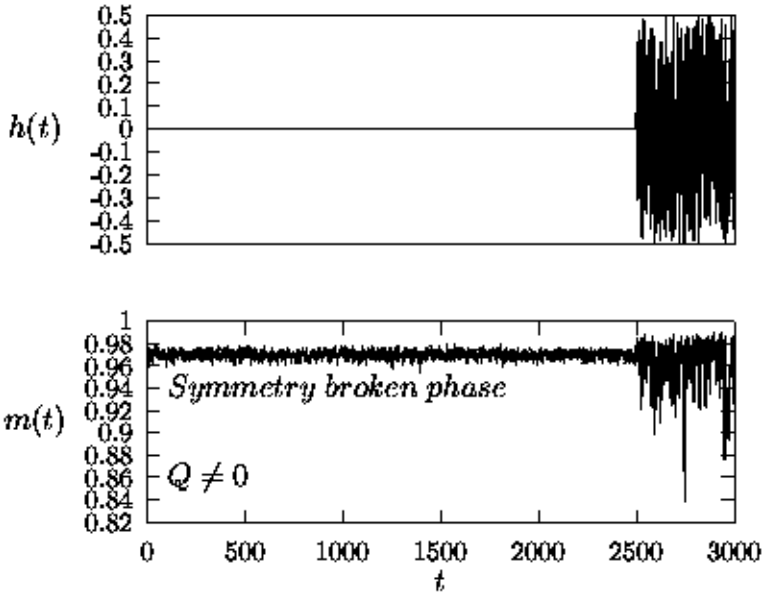

(d)
$T = 0.40J/K_B$
$h_0 = 2.0J$
$P(Q)$
Q

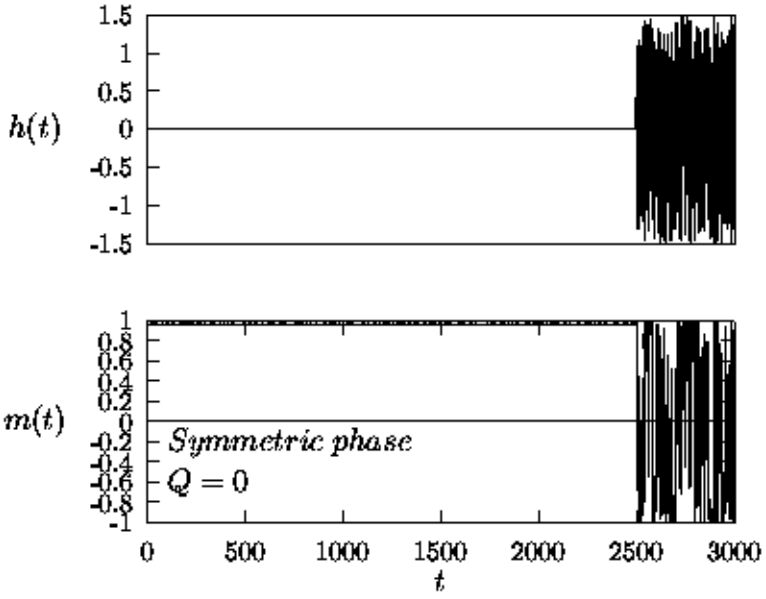

(a)
$T = 1.48J/K_B$
$h_0 = 0.3J$
$P(Q)$
Q
(b)
$T = 1.50J/K_B$
$h_0 = 0.3J$
$P(Q)$
Q
(c)
$T = 1.55J/K_B$
$h_0 = 0.3J$
$P(Q)$
Q

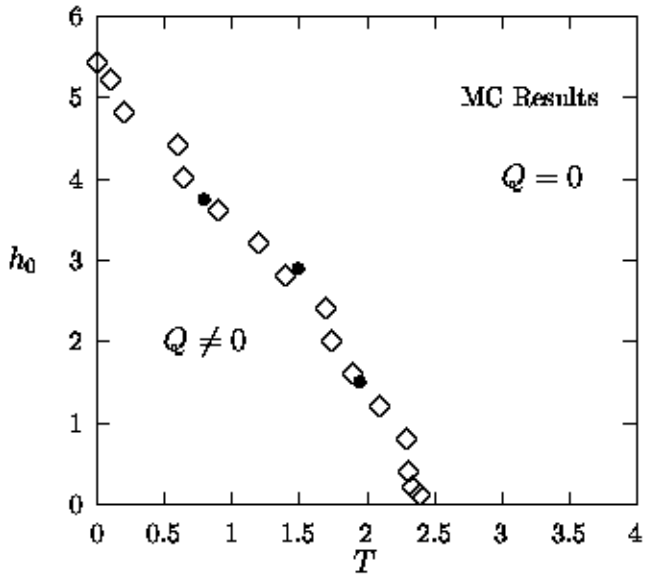

0.005
0.0045
0.004
0.0035
0.003
$P_\tau(\tau_\tau)$
0.0025
$[MCSS^{-1}]$
0.002
0.0015
0.001
0.0005
0
0
1000
2000
3000
4000
5000
6000
$\tau_\tau[MCSS]$
$T = 0.30J/K_B$
$h_0 = 2.0J$

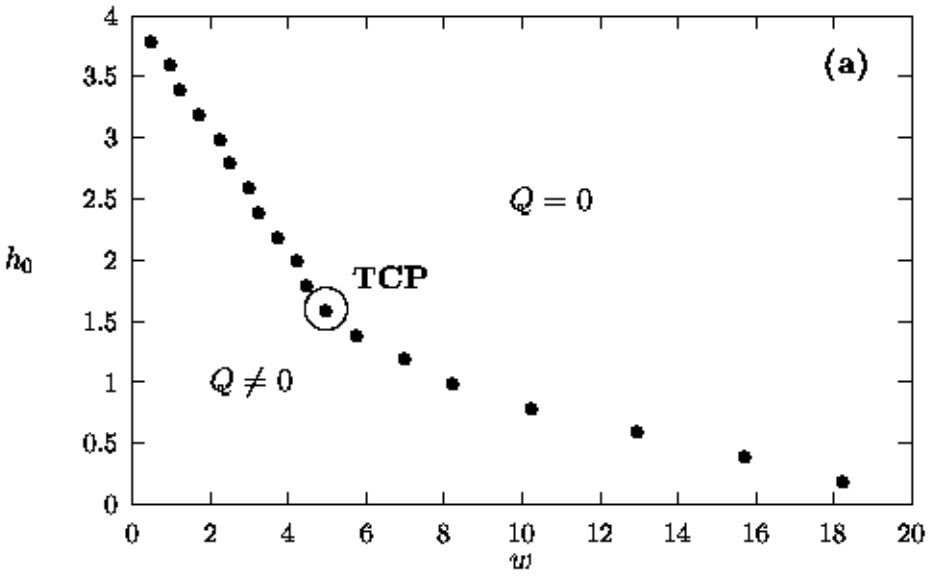

T=1.0J
(a)
T=0.525J
(b)
T=0.5J
(c)
T=0.3J
(d)
P(Q)
Q

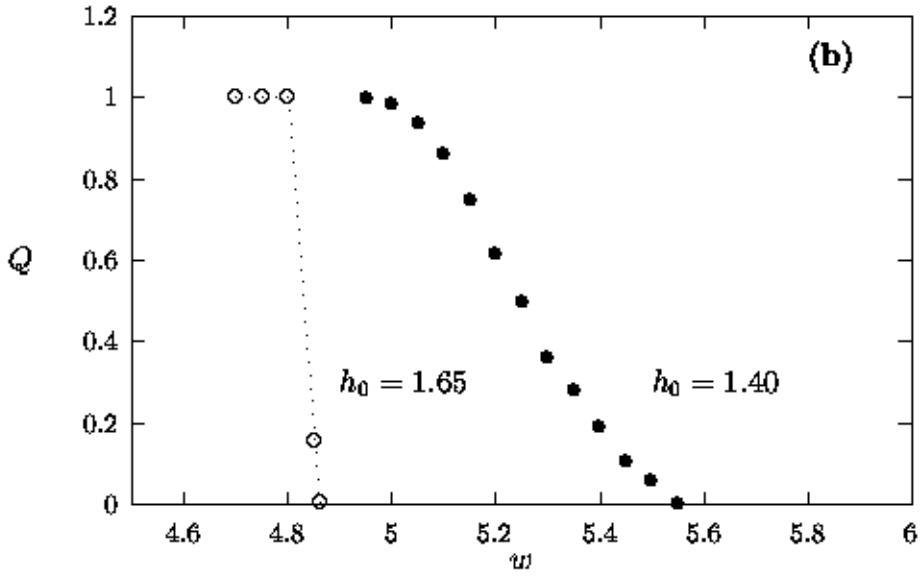

T=0.5J
(a)
T=0.4J
(b)
T=0.375J
(c)
T=0.35J
(d)
P(Q)
Q

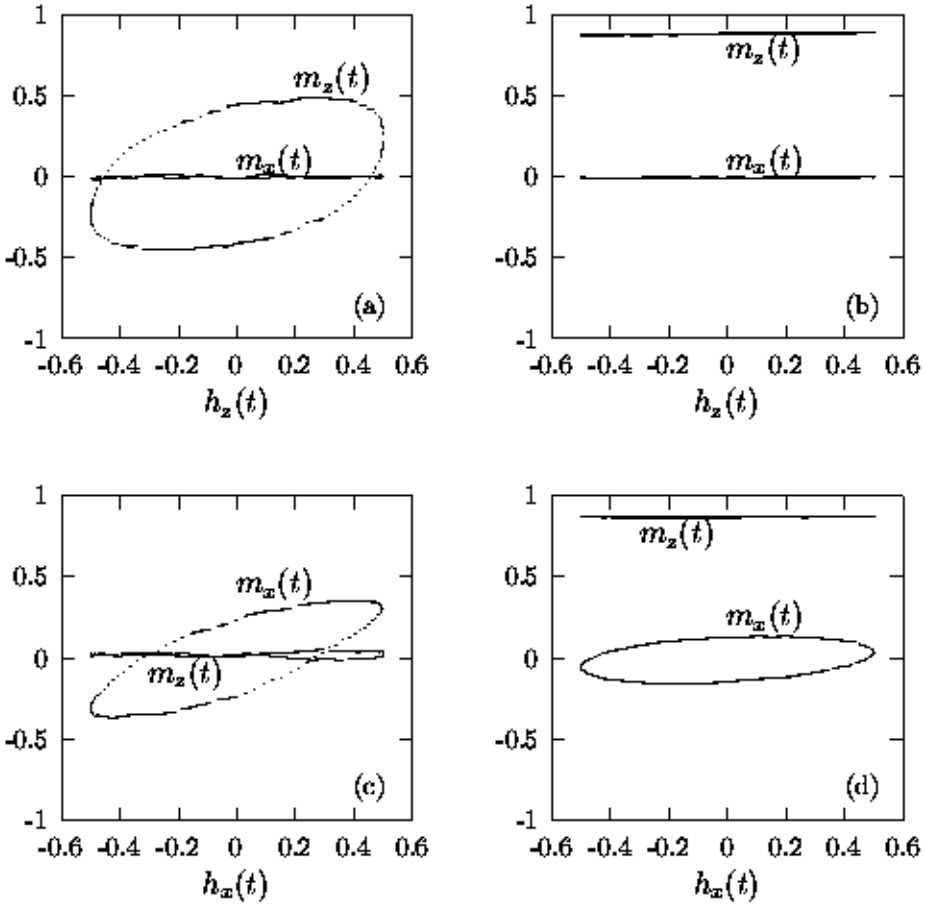

h(t)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
m(t)
1
0.98
0.96
0.94
0.92
0.9
0.88
0.86
0.84
0.82
Symmetry broken phase
$Q \neq 0$
0
500
1000
1500
2000
2500
3000
t

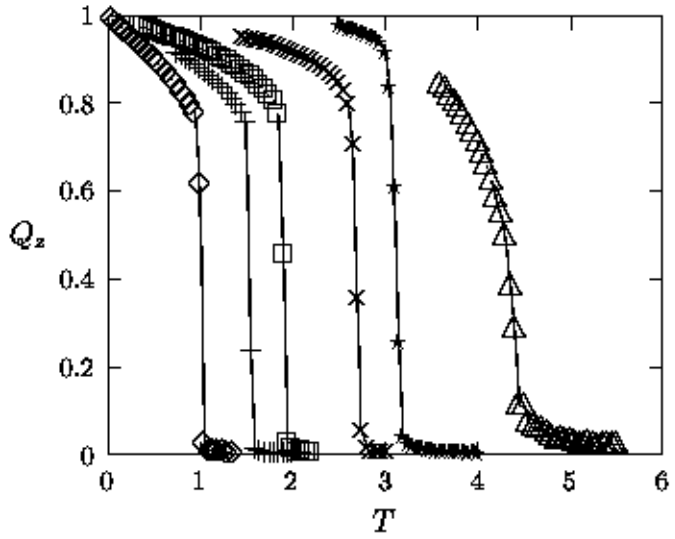

h(t)
1.5
1
0.5
0
-0.5
-1
-1.5
m(t)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
Symmetric phase
Q = 0
0
500
1000
1500
2000
2500
3000
t

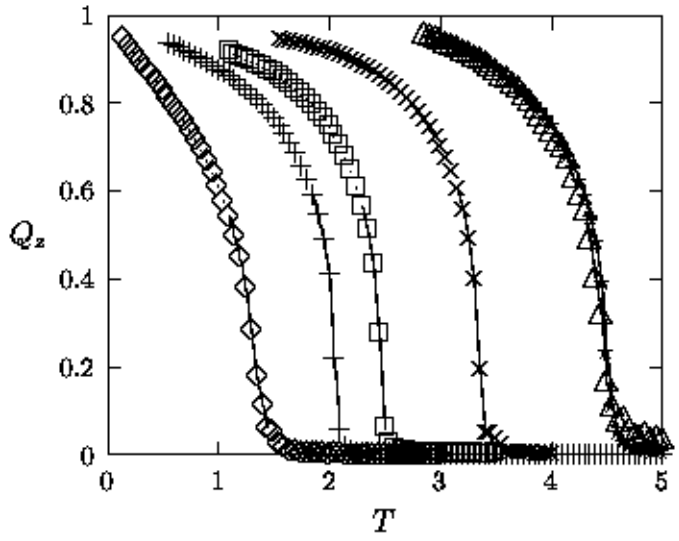

MC Results
$Q = 0$
$Q \neq 0$
h_0
T
0
1
2
3
4
5
6
0
0.5
1
1.5
2
2.5
3
3.5
4

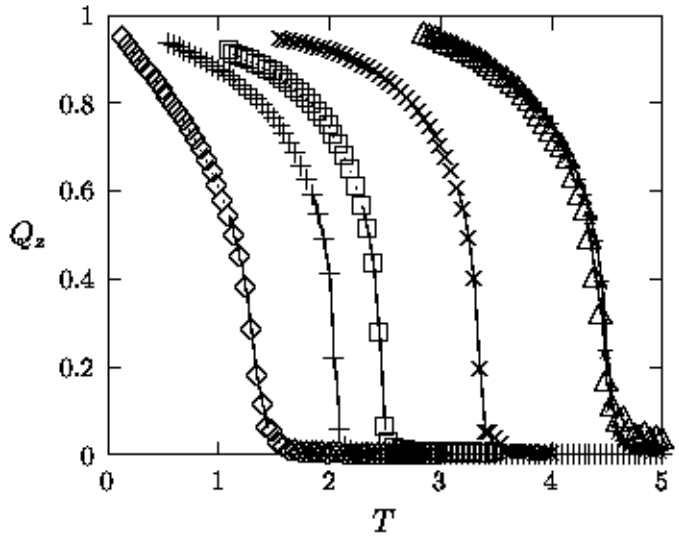

(a)
$Q = 0$
TCP
$Q \neq 0$
h_0
w
0
0.5
1
1.5
2
2.5
3
3.5
4
0
2
4
6
8
10
12
14
16
18
20

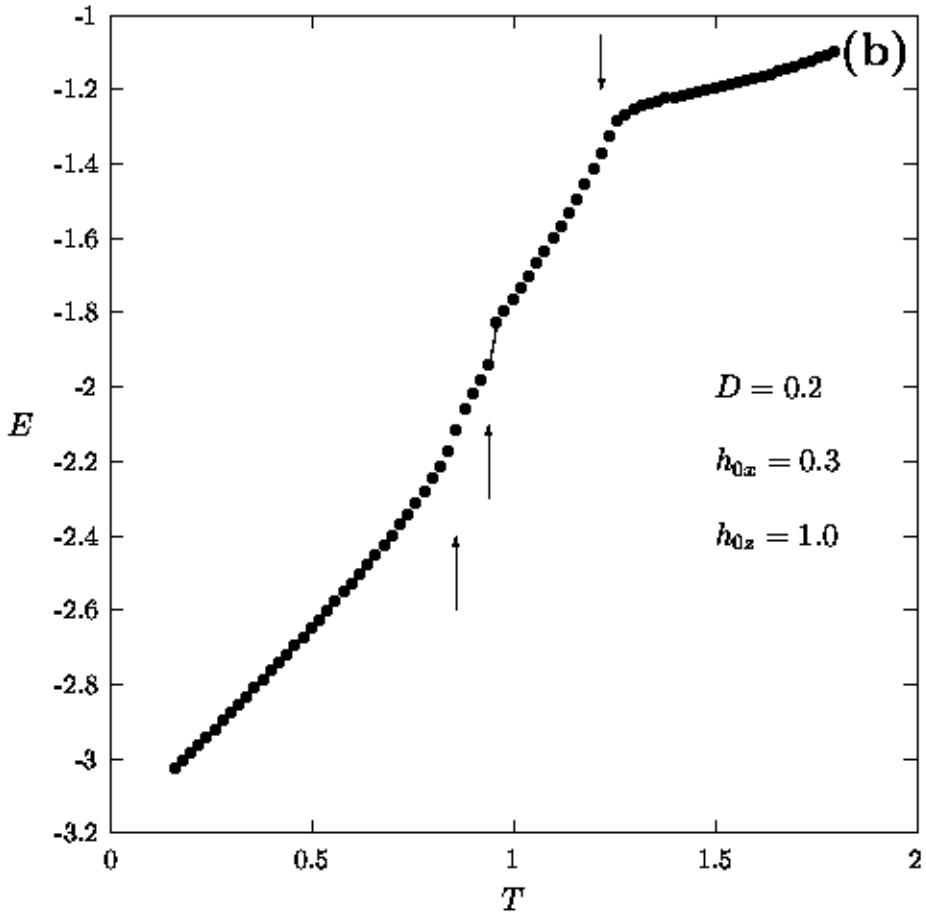

(b)
$h_0 = 1.65$
$h_0 = 1.40$
Q
w
1.2
1
0.8
0.6
0.4
0.2
0
4.6
4.8
5
5.2
5.4
5.6
5.8
6

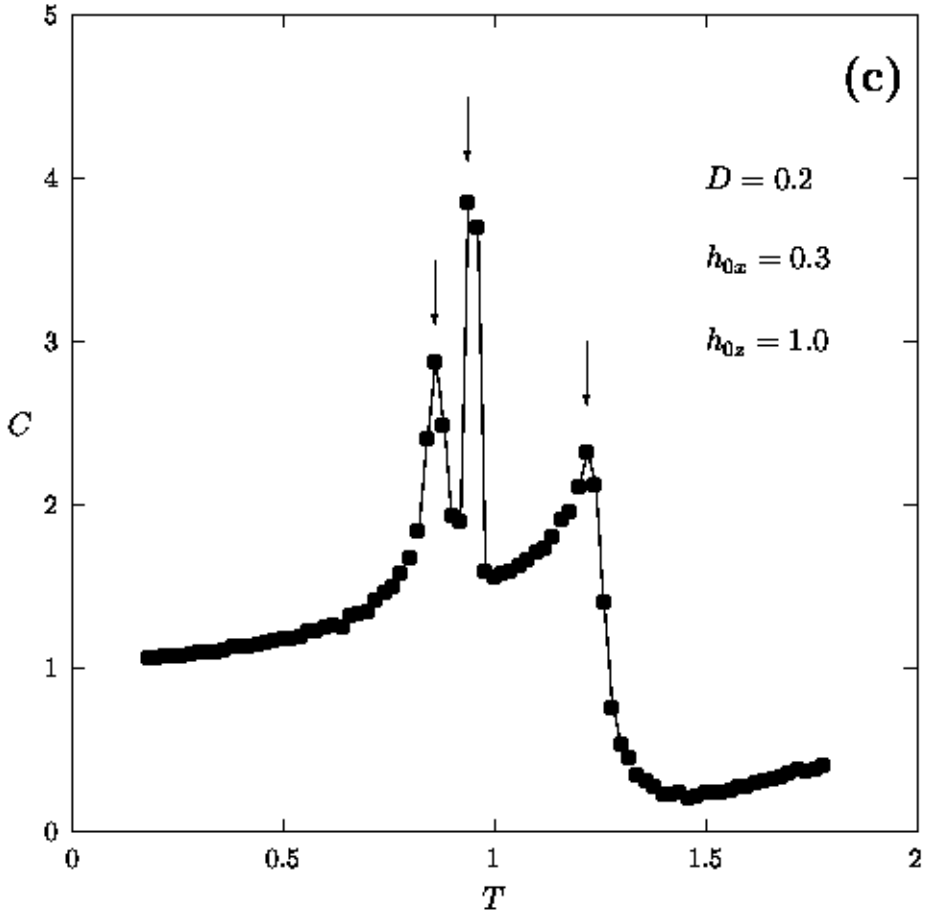

1
0.5
0
-0.5
-1
-0.6 -0.4 -0.2 0 0.2 0.4 0.6
$h_z(t)$
$m_z(t)$
$m_x(t)$
(a)
(b)
$h_x(t)$
(c)
(d)

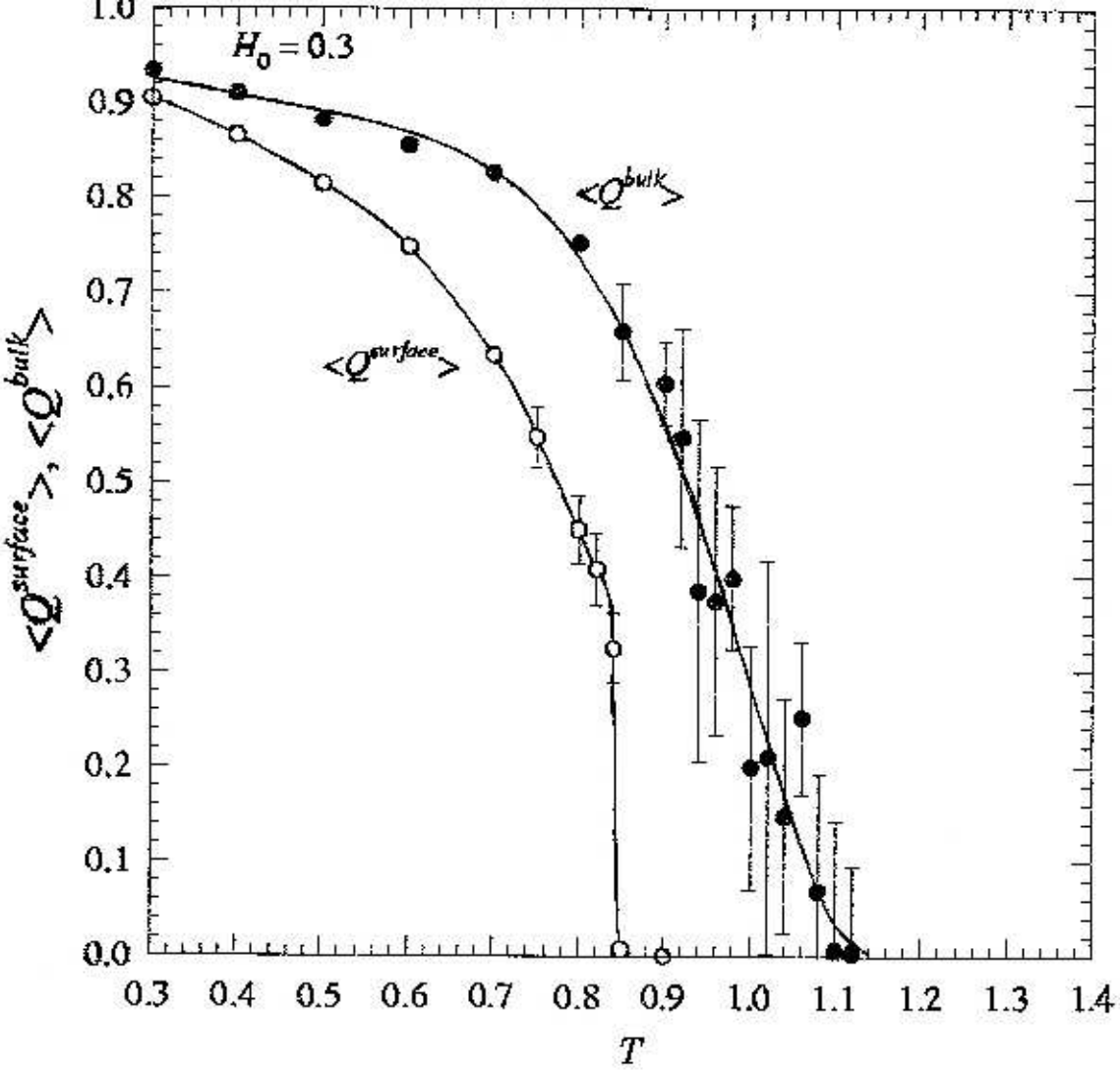

1
0.8
0.6
0.4
0.2
0
Q_z
0
1
2
3
4
5
6
T

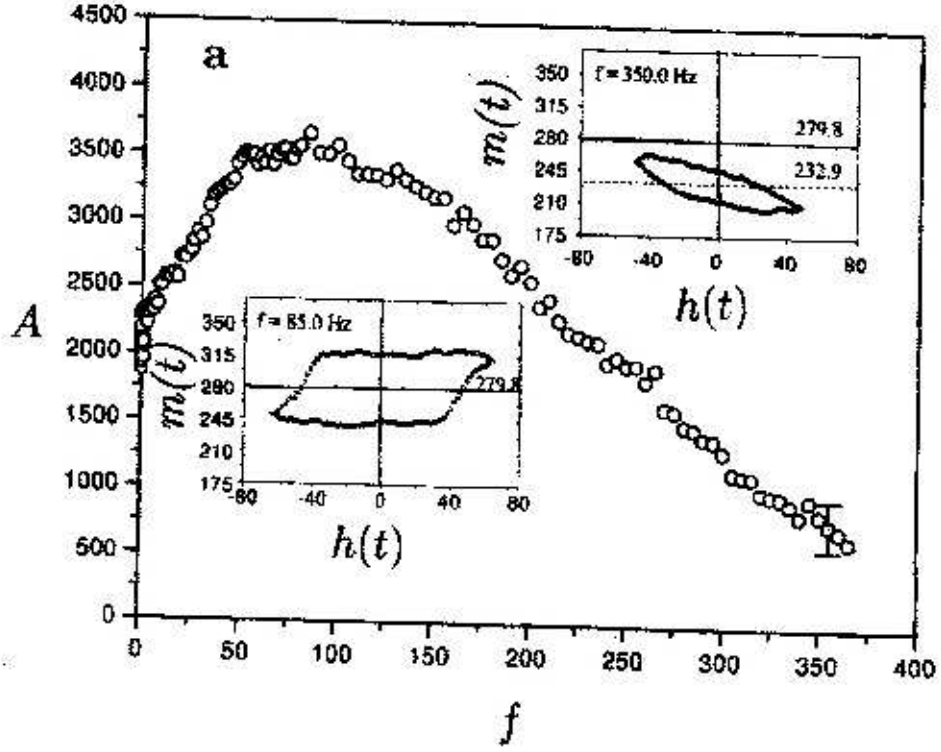

1
0.8
0.6
0.4
0.2
0
Q_z
0
1
2
3
4
5
T

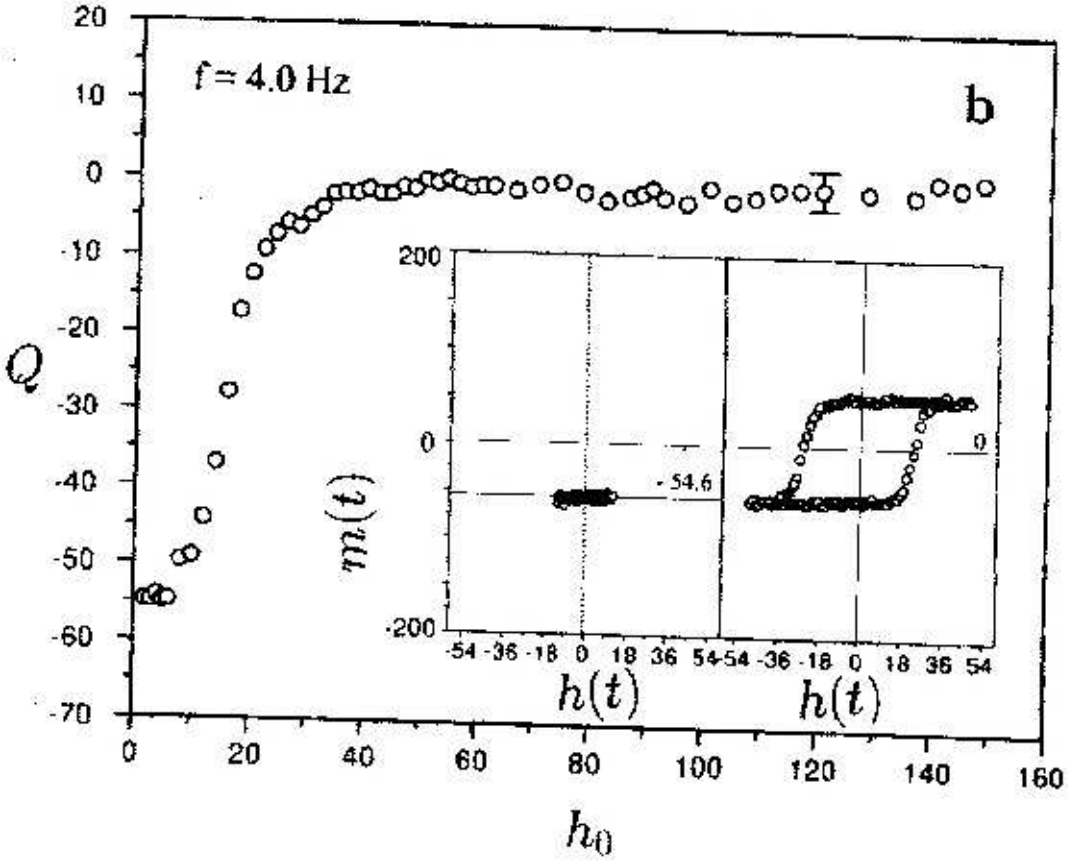

1
0.8
0.6
0.4
0.2
0
Q_z
0
1
2
3
4
5
T


(b)
$D = 0.2$
$h_{0x} = 0.3$
$h_{0z} = 1.0$
E
T
-1
-1.2
-1.4
-1.6
-1.8
-2
-2.2
-2.4
-2.6
-2.8
-3
-3.2
0
0.5
1
1.5
2


(c)
$D = 0.2$
$h_{0x} = 0.3$
$h_{0z} = 1.0$
C
T
0
1
2
3
4
5
0
0.5
1
1.5
2


$H_0 = 0.3$
$\langle Q^{bulk} \rangle$
$\langle Q^{surface} \rangle$
$\langle Q^{surface} \rangle$, $\langle Q^{bulk} \rangle$
T
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4


a
A
f
m(t)
h(t)
f = 350.0 Hz
279.8
232.9
f = 85.0 Hz
279.8


b
f = 4.0 Hz
Q
h_0
m(t)
h(t)
-54.6
0